

Populationsdynamik

1 Erste Vorgehensweise

```
Remove["Global`*"]
```

Bemerkung zur Populationsmatrix B

```
B = {{3.5, -25}, {0.05, 0.5}};  
B // MatrixForm
```

$$\begin{pmatrix} 3.5 & -25 \\ 0.05 & 0.5 \end{pmatrix}$$

```
B = {{7 / 2, -25}, {5 / 100, 5 / 10}};  
B // MatrixForm
```

$$\begin{pmatrix} \frac{7}{2} & -25 \\ \frac{1}{20} & \frac{1}{2} \end{pmatrix}$$

```
Eigenvalues[B]
```

```
{3, 1}
```

```
Eigenvectors[B]
```

```
{{50, 1}, {10, 1}}
```

a) Negative Resultate

```

x[1]={1,1}; B={{3.5,-25},{0.05,0.5}};
x[n_]:=B.x[n-1];
z[k_]:=Prepend[Append[x[k],x[k][[1]]/x[k][[2]]],k];
Prepend[Table[z[k],{k,1,20}],{"Nr.", "x", "y", "Quot.
x/y"}]//N//MatrixForm

```

Nr.	x	y	Quot. x/y
1.	1.	1.	1.
2.	-21.5	0.55	-39.0909
3.	-89.	-0.8	111.25
4.	-291.5	-4.85	60.1031
5.	-899.	-17.	52.8824
6.	-2721.5	-53.45	50.9167
7.	-8189.	-162.8	50.301
8.	-24591.5	-490.85	50.0998
9.	-73799.	-1475.	50.0332
10.	-221422.	-4427.45	50.0111
11.	-664289.	-13284.8	50.0037
12.	-1.99289×10^6	-39856.9	50.0012
13.	-5.9787×10^6	-119573.	50.0004
14.	-1.79361×10^7	-358721.	50.0001
15.	-5.38084×10^7	-1.07617×10^6	50.
16.	-1.61425×10^8	-3.2285×10^6	50.
17.	-4.84276×10^8	-9.68551×10^6	50.
18.	-1.45283×10^9	-2.90565×10^7	50.
19.	-4.35848×10^9	-8.71696×10^7	50.
20.	-1.30754×10^{10}	-2.61509×10^8	50.

```

x[1]={5,1}; B={{3.5,-25},{0.05,0.5}};
x[n_]:=B.x[n-1];
z[k_]:=Prepend[Append[x[k],x[k][[1]]/x[k][[2]]],k];
Prepend[Table[z[k],{k,1,20}],{"Nr.", "x", "y", "Quot.
x/y"}]//N//MatrixForm

```

Nr.	x	y	Quot. x/y
1.	5.	1.	5.
2.	-7.5	0.75	-10.
3.	-45.	-2.08167×10^{-17}	2.16173×10^{18}
4.	-157.5	-2.25	70.
5.	-495.	-9.	55.
6.	-1507.5	-29.25	51.5385
7.	-4545.	-90.	50.5
8.	-13657.5	-272.25	50.1653
9.	-40995.	-819.	50.0549
10.	-123008.	-2459.25	50.0183
11.	-369045.	-7380.	50.0061
12.	-1.10716×10^6	-22142.3	50.002
13.	-3.3215×10^6	-66429.	50.0007
14.	-9.96451×10^6	-199289.	50.0002
15.	-2.98935×10^7	-597870.	50.0001
16.	-8.96807×10^7	-1.79361×10^6	50.
17.	-2.69042×10^8	-5.38084×10^6	50.
18.	-8.07126×10^8	-1.61425×10^7	50.
19.	-2.42138×10^9	-4.84276×10^7	50.
20.	-7.26413×10^9	-1.45283×10^8	50.

b) Stabilität

```

x[1]={10,1}; B={{3.5,-25},{0.05,0.5}};
x[n_]:=B.x[n-1];
z[k_]:=Prepend[Append[x[k],x[k][[1]]/x[k][[2]]],k];
Prepend[Table[z[k],{k,1,20}],{"Nr.", "x", "y", "Quot.
x/y"}]//N//MatrixForm

```

```

( Nr.  x    y    Quot. x/y )
1.  10.  1.  10.
2.  10.  1.  10.
3.  10.  1.  10.
4.  10.  1.  10.
5.  10.  1.  10.
6.  10.  1.  10.
7.  10.  1.  10.
8.  10.  1.  10.
9.  10.  1.  10.
10. 10.  1.  10.
11. 10.  1.  10.
12. 10.  1.  10.
13. 10.  1.  10.
14. 10.  1.  10.
15. 10.  1.  10.
16. 10.  1.  10.
17. 10.  1.  10.
18. 10.  1.  10.
19. 10.  1.  10.
20. 10.  1.  10.

```

c) Zunahme, positive Resultate, Quotient konvergiert von unten her

```

x[1]={25,1}; B={{3.5,-25},{0.05,0.5}};
x[n_]:=B.x[n-1];
z[k_]:=Prepend[Append[x[k],x[k][[1]]/x[k][[2]]],k];
Prepend[Table[z[k],{k,1,20}],{"Nr.", "x", "y", "Quot.
x/y"}]//N//MatrixForm

```

Nr.	x	y	Quot. x/y
1.	25.	1.	25.
2.	62.5	1.75	35.7143
3.	175.	4.	43.75
4.	512.5	10.75	47.6744
5.	1525.	31.	49.1935
6.	4562.5	91.75	49.7275
7.	13675.	274.	49.9088
8.	41012.5	820.75	49.9695
9.	123025.	2461.	49.9898
10.	369063.	7381.75	49.9966
11.	1.10718×10^6	22144.	49.9989
12.	3.32151×10^6	66430.8	49.9996
13.	9.96453×10^6	199291.	49.9999
14.	2.98936×10^7	597872.	50.
15.	8.96807×10^7	1.79361×10^6	50.
16.	2.69042×10^8	5.38084×10^6	50.
17.	8.07126×10^8	1.61425×10^7	50.
18.	2.42138×10^9	4.84276×10^7	50.
19.	7.26413×10^9	1.45283×10^8	50.
20.	2.17924×10^{10}	4.35848×10^8	50.

d) Proportionale Zunahme

```

x[1]={50,1}; B={{3.5,-25},{0.05,0.5}};
x[n_]:=B.x[n-1];
z[k_]:=Prepend[Append[x[k],x[k][[1]]/x[k][[2]]],k];
Prepend[Table[z[k],{k,1,20}],{"Nr.", "x", "y", "Quot.
x/y"}]//N//MatrixForm

```

Nr.	x	y	Quot. x/y
1.	50.	1.	50.
2.	150.	3.	50.
3.	450.	9.	50.
4.	1350.	27.	50.
5.	4050.	81.	50.
6.	12150.	243.	50.
7.	36450.	729.	50.
8.	109350.	2187.	50.
9.	328050.	6561.	50.
10.	984150.	19683.	50.
11.	2.95245×10^6	59049.	50.
12.	8.85735×10^6	177147.	50.
13.	2.65721×10^7	531441.	50.
14.	7.97162×10^7	1.59432×10^6	50.
15.	2.39148×10^8	4.78297×10^6	50.
16.	7.17445×10^8	1.43489×10^7	50.
17.	2.15234×10^9	4.30467×10^7	50.
18.	6.45701×10^9	1.2914×10^8	50.
19.	1.9371×10^{10}	3.8742×10^8	50.
20.	5.81131×10^{10}	1.16226×10^9	50.

e) Zunahme, positive Resultate, Quotient konvergiert von oben her

```

x[1]={70,1}; B={{3.5,-25},{0.05,0.5}};
x[n_]:=B.x[n-1];
z[k_]:=Prepend[Append[x[k],x[k][[1]]/x[k][[2]]],k];
Prepend[Table[z[k],{k,1,20}],{"Nr.", "x", "y", "Quot.
x/y"}]//N//MatrixForm

```

Nr.	x	y	Quot. x/y
1.	70.	1.	70.
2.	220.	4.	55.
3.	670.	13.	51.5385
4.	2020.	40.	50.5
5.	6070.	121.	50.1653
6.	18220.	364.	50.0549
7.	54670.	1093.	50.0183
8.	164020.	3280.	50.0061
9.	492070.	9841.	50.002
10.	1.47622×10^6	29524.	50.0007
11.	4.42867×10^6	88573.	50.0002
12.	1.3286×10^7	265720.	50.0001
13.	3.98581×10^7	797161.	50.
14.	1.19574×10^8	2.39148×10^6	50.
15.	3.58723×10^8	7.17445×10^6	50.
16.	1.07617×10^9	2.15234×10^7	50.
17.	3.2285×10^9	6.45701×10^7	50.
18.	9.68551×10^9	1.9371×10^8	50.
19.	2.90565×10^{10}	5.81131×10^8	50.
20.	8.71696×10^{10}	1.74339×10^9	50.

```
x[1]={100,1}; B={{3.5, -25},{0.05,0.5}};
x[n_]:= B.x[n-1];
z[k_]:=Prepend[Append[x[k],x[k][[1]]/x[k][[2]]],k];
Prepend[Table[z[k],{k,1,20}],{"Nr.", "x", "y", "Quot.
x/y"}]//N//MatrixForm
```

Nr.	x	y	Quot. x/y
1.	100.	1.	100.
2.	325.	5.5	59.0909
3.	1000.	19.	52.6316
4.	3025.	59.5	50.8403
5.	9100.	181.	50.2762
6.	27325.	545.5	50.0917
7.	82000.	1639.	50.0305
8.	246025.	4919.5	50.0102
9.	738100.	14761.	50.0034
10.	2.21433×10^6	44285.5	50.0011
11.	6.643×10^6	132859.	50.0004
12.	1.9929×10^7	398580.	50.0001
13.	5.97871×10^7	1.19574×10^6	50.
14.	1.79361×10^8	3.58723×10^6	50.
15.	5.38084×10^8	1.07617×10^7	50.
16.	1.61425×10^9	3.2285×10^7	50.
17.	4.84276×10^9	9.68551×10^7	50.
18.	1.45283×10^{10}	2.90565×10^8	50.
19.	4.35848×10^{10}	8.71696×10^8	50.
20.	1.30754×10^{11}	2.61509×10^9	50.

2 Andere Vorgehensweise

```
Remove[x]
```

```
x[1] = 25; y[1] = 1;
x[n_] := (B.{x[n-1], y[n-1]})[[1]];
y[n_] := (B.{x[n-1], y[n-1]})[[2]];
```

```
{x[1], y[1]}
```

```
{25, 1}
```

```
{x[2], y[2]} // N
```

```
{62.5, 1.75}
```

```
Table[{x[k], y[k]}, {k, 1, 15}] // N // MatrixForm (* Langsamer *)
```

25.	1.
62.5	1.75
175.	4.
512.5	10.75
1525.	31.
4562.5	91.75
13675.	274.
41012.5	820.75
123025.	2461.
369063.	7381.75
1.10718×10^6	22144.
3.32151×10^6	66430.8
9.96453×10^6	199291.
2.98936×10^7	597872.
8.96807×10^7	1.79361×10^6