Eine Bootstrap-Anwendung Schritt für Schritt

Mit *Mathematica*

Fall 1 resp. Beispiel 1: Generierung einer Menge von Zahlen als Messwerte

```
Remove["Global`*"]
```

Die Funktion "Random" (Wahl einer Zufallszahl zwischen 0 und 1)

```
Random[]
```

```
0.364739
```

Wahl einer ganzen Zufallszahl zwischen 1 und 12

```
Random[Integer, {1, 12}]
```

```
1
```

Wahl von 20 ganzen Zufallszahlen zwischen 1 und 12

```
Table[Random[Integer, {1, 12}], {n, 1, 20}]
```

```
{3, 6, 1, 6, 5, 10, 11, 11, 11, 6, 8, 12, 10, 5, 10, 12, 7, 2, 7, 10}
```

Wahl von 50 Zufallszahlen zwischen -10 und 10

```
M = Table[20 Random[] - 10, {n, 1, 50}]
```

```
{-8.04173, 9.20101, -5.06533, 8.19742, 8.06977, -8.91607, 1.25747, 2.95609, 1.37916,
 -2.25305, 5.28348, -7.44429, -7.59717, 3.49109, 7.99958, -8.01813, 1.36497,
 -4.43999, -3.25404, 0.0386037, 0.759815, 1.98081, 6.52506, -7.25618, -1.19846,
 2.7798, 1.59039, -5.4536, 0.73177, 1.69587, -9.66708, 1.59031, 9.35261, -6.05108,
 -4.95056, -0.965404, 6.94978, 0.457826, -2.95014, -2.94728, -4.41519, -5.10219,
 -9.6961, 7.01412, 4.82499, 2.917, -6.22116, 4.2703, -3.97655, -9.86279}
```

Kontrolle:

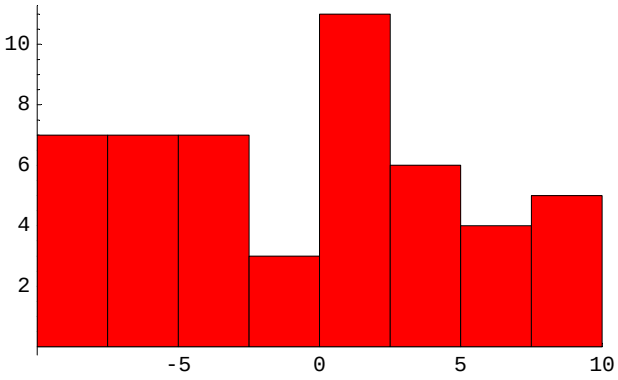
```
M
```

```
{-8.04173, 9.20101, -5.06533, 8.19742, 8.06977, -8.91607, 1.25747, 2.95609, 1.37916,
 -2.25305, 5.28348, -7.44429, -7.59717, 3.49109, 7.99958, -8.01813, 1.36497,
 -4.43999, -3.25404, 0.0386037, 0.759815, 1.98081, 6.52506, -7.25618, -1.19846,
 2.7798, 1.59039, -5.4536, 0.73177, 1.69587, -9.66708, 1.59031, 9.35261, -6.05108,
 -4.95056, -0.965404, 6.94978, 0.457826, -2.95014, -2.94728, -4.41519, -5.10219,
 -9.6961, 7.01412, 4.82499, 2.917, -6.22116, 4.2703, -3.97655, -9.86279}
```

Fall 1 resp. Beispiel 1: Bearbeitung der Menge von Messwerten

Histogramm und Mittelwert von M

```
<< Graphics`Graphics`;
Histogram[M];
M // Mean
```




```
-0.661289
```

Die 10-te Zahl aus M

```
z1 = M[[10]]
```

```
-2.25305
```

Kontrolle:

```
z1
```

```
-2.25305
```

Die n-te Zahl aus M, n = ganze Zufallszahl zwischen 1 und 50 (50 = |M|)

```
n1 = Random[Integer, {1, 50}]; z2 = M[[n1]]; {n1, z2}
```

```
{29, 0.73177}
```

Kontrolle:

```
z2
```

```
0.73177
```

Eine Funktion u, die mit einer Nummer j eine Menge von 50 Zufallszahlen aus M so auswählt wie es eben geschehen ist (Exemplar Nummer j mit 50 Zahlen aus M)

```
u[j_] := Table[M[[Random[Integer, {1, 50}]]], {k, 1, 50}]
```

```
tWork1 = u[1]
```

```
{-9.66708, 9.20101, -8.04173, 9.20101, 1.59031, -7.59717, -2.25305, -2.95014,
 -7.59717, 0.759815, -5.06533, -8.04173, -2.95014, 5.28348, 6.94978, -7.44429,
 -7.25618, -3.97655, 9.20101, -9.6961, 0.457826, 1.59039, 1.59039, 1.37916,
 -4.43999, -1.19846, 1.59031, -7.44429, 1.36497, -3.97655, 4.82499, 9.20101,
 -7.25618, 0.457826, -3.25404, 2.917, 8.06977, 8.19742, 1.98081, -3.97655, -9.6961,
 7.01412, 2.95609, 8.06977, 2.917, -0.965404, -7.59717, 1.25747, 8.19742, -2.95014}
```

Kontrolle:

```
tWork1
```

```
{-9.66708, 9.20101, -8.04173, 9.20101, 1.59031, -7.59717, -2.25305, -2.95014,
 -7.59717, 0.759815, -5.06533, -8.04173, -2.95014, 5.28348, 6.94978, -7.44429,
 -7.25618, -3.97655, 9.20101, -9.6961, 0.457826, 1.59039, 1.59039, 1.37916,
 -4.43999, -1.19846, 1.59031, -7.44429, 1.36497, -3.97655, 4.82499, 9.20101,
 -7.25618, 0.457826, -3.25404, 2.917, 8.06977, 8.19742, 1.98081, -3.97655, -9.6961,
 7.01412, 2.95609, 8.06977, 2.917, -0.965404, -7.59717, 1.25747, 8.19742, -2.95014}
```

Kontrolle der Anzahl der gewählten Zahlen

```
Length[u[1]]
```

```
50
```

Mit u[j], j = 1 bis 80, wird in eine Tabelle mit dem Namen \glqq t\Grqq 80 mal der Mittelwert von 50 Zufallszahlen aus M geschrieben: Das sind jetzt Mittelwerte von 80 Bootstrap--Kopien!

```
tWork2 = Table[Mean[u[j]], {j, 1, 80}]
```

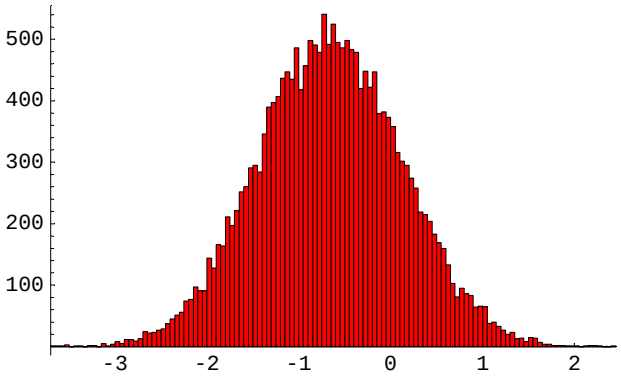
```
{-0.434433, -1.74993, -0.662409, -1.72697, -0.750001, -0.262258, -1.12953, -0.808541,
 -2.76731, 0.244253, -0.872622, -1.32, -0.168366, -0.417323, -1.01466, -0.684575,
 0.735755, -0.294019, -0.163264, -0.949225, -1.06264, 0.222416, 0.581721,
 -0.701841, 0.622925, -0.0480309, -1.81834, 0.305353, -1.22708, -0.437906,
 -1.07985, -1.22379, -0.150394, -1.09496, 0.302957, -0.233302, -0.122682,
 -0.468656, -1.57165, -0.341909, -1.67767, -1.41352, -0.937899, 0.642327,
 -1.3351, -1.7091, -0.546987, -1.37958, -0.282523, -0.194144, -0.891183,
 -0.906254, -1.6141, -0.945349, -1.71538, -0.765694, -0.996419, -0.54514,
 -0.353011, -2.20551, -0.133752, -1.99811, 0.415847, -0.749793, -1.81067,
 -2.73591, -1.72515, -1.41604, -1.2084, -1.54913, -0.50211, -1.9884, -0.648133,
 -0.638849, -0.286659, -1.2501, -1.23166, -0.163085, -0.961458, 0.187376}
```

```
tWork2
```

```
{-0.434433, -1.74993, -0.662409, -1.72697, -0.750001, -0.262258, -1.12953, -0.808541,
 -2.76731, 0.244253, -0.872622, -1.32, -0.168366, -0.417323, -1.01466, -0.684575,
 0.735755, -0.294019, -0.163264, -0.949225, -1.06264, 0.222416, 0.581721,
 -0.701841, 0.622925, -0.0480309, -1.81834, 0.305353, -1.22708, -0.437906,
 -1.07985, -1.22379, -0.150394, -1.09496, 0.302957, -0.233302, -0.122682,
 -0.468656, -1.57165, -0.341909, -1.67767, -1.41352, -0.937899, 0.642327,
 -1.3351, -1.7091, -0.546987, -1.37958, -0.282523, -0.194144, -0.891183,
 -0.906254, -1.6141, -0.945349, -1.71538, -0.765694, -0.996419, -0.54514,
 -0.353011, -2.20551, -0.133752, -1.99811, 0.415847, -0.749793, -1.81067,
 -2.73591, -1.72515, -1.41604, -1.2084, -1.54913, -0.50211, -1.9884, -0.648133,
 -0.638849, -0.286659, -1.2501, -1.23166, -0.163085, -0.961458, 0.187376}
```

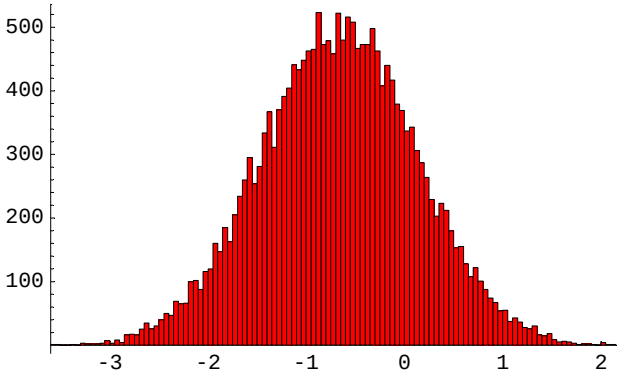
Jetzt wird 20'000 mal der Mittelwert von 50 Zufallszahlen aus M in eine Tabelle geschrieben und damit ein Histogramm der Mittelwerte erstellt

```
t = Table[Mean[u[j]], {j, 1, 20000}]; Histogram[t];
```



Wir wiederholen das Programm und erstellen nochmals ein Histogramm (andere Bootstrapkopien, anderes Histogramm, ähnliche Form, aber verschoben...)

```
t = Table[Mean[u[j]], {j, 1, 20000}]; Histogram[t];
```

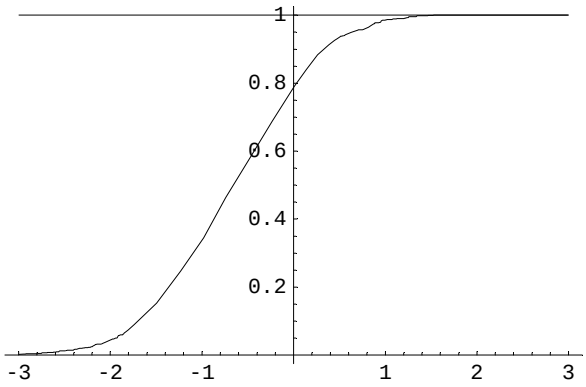


Die Form des Histogramms bleibt bei 20'000 mal "bootstrapen" beim zweiten Mal etwa ähnlich. Daher kann man einer daraus gewonnenen Verteilungsfunktion vertrauen.

Erstellung einer Verteilungsfunktion auf der Grundlage von Bootstrap-Kopien (hier mit 1000 Kopien)

```
<< Statistics`DataManipulation`

ttt = Sort[Table[Mean[u[j]], {j, 1, 1000}]];
freq = Union[Frequencies[ttt]];
F[x_] := Apply[Plus, Table[Take[freq, Length[Select[Table[
          freq[[k1, 2]], {k1, 1, Length[freq]}], # ≤ x &]]][[k1]][[1]],
    {k1, 1, Length[Take[freq, Length[Select[Table[freq[[k, 2]],
          {k, 1, Length[freq]}], # ≤ x &]]]]}]] / Length[ttt] ;
Plot[{F[x], 1}, {x, -3, 3}];
```




```
freq = Union[Frequencies[t]];
(* Ergibt 20'000 Zahlenpaare! Diese werden des Umfangs
  wegen nicht ausgegeben. Daher der ";" an Ende des Befehls. *)
```

Fall 2 resp. Beispiel 2: Einlesen und bearbeiten von Messwerten

Hier wird erst eine neue Messreihe erzeugt.

```
Remove["Global`*"]
```

Einlesen resp. Erzeugung der Messwerte und Setzung der Parameter

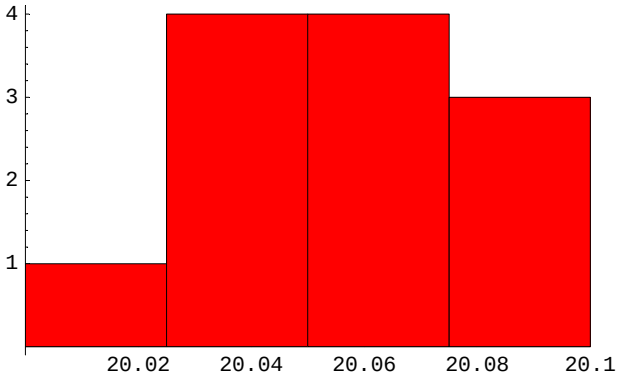
```
nN = 12;
nHistogram = 20000;
nBootstrap = 1000;
M1 = Table[Random[] / 10 + 20, {n, 1, nN}];
```

Bearbeitung mit Plug-in (Bootstrapping)

```
<< Graphics`Graphics`;
<< Statistics`DataManipulation`
M1 = Table[Random[] / 10 + 20, {n, 1, nN}];
Print["Messwerte:  ", M1];
Print["Mittelwert: ", M1 // Mean];
Histogram[M1];
u1[j_] := Table[M1[[Random[Integer, {1, nN}]]], {k, 1, 50}];
Print["Kontrolle Anzahl:  ", Length[u1[1]] == nN];
Print[nBootstrap, " mal bootstrapen"];
t1 = Table[Mean[u1[j]], {j, 1, nBootstrap}]; Histogram[t1];
Print["Nochmals ", nBootstrap, " mal bootstrapen"];
t1 = Table[Mean[u1[j]], {j, 1, nBootstrap}]; Histogram[t1];
Print[nHistogram, " mal bootstrapen"];
t1 = Table[Mean[u1[j]], {j, 1, nHistogram}]; Histogram[t1];
Print["Nochmals ", nHistogram, " mal bootstrapen"];
t1 = Table[Mean[u1[j]], {j, 1, nHistogram}]; Histogram[t1];
ttt1 = Sort[Table[Mean[u1[j]], {j, 1, nBootstrap}]];
Print[nBootstrap, " Bootstrap-Kopien der Mittelwerte= ", ttt1];
MeanMin = Min[ttt1];
MeanMax = Max[ttt1];
Print["Mittelwert Min. bis Max.  = ",  "[", MeanMin, ", ", MeanMax, "]"];
freq = Union[Frequencies[ttt1]];
F[x_] := Apply[Plus, Table[Take[freq, Length[Select[Table[
          freq[[k1, 2]], {k1, 1, Length[freq]}], # ≤ x &]]][[k1]][[1]],
    {k1, 1, Length[Take[freq, Length[Select[Table[freq[[k, 2]],
          {k, 1, Length[freq]}], # ≤ x &]]]]}]]  / Length[ttt1] ;
Plot[{F[x], 1}, {x, MeanMin, MeanMax}];
```

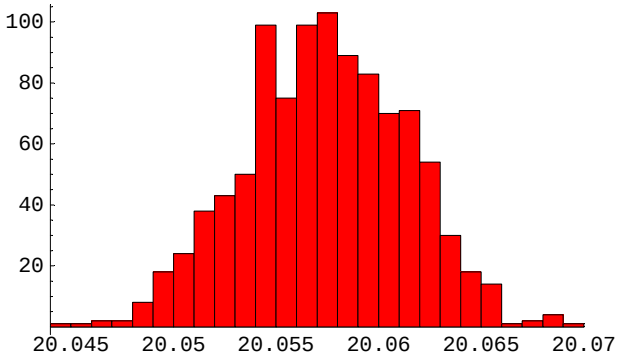
```
Messwerte:  {20.0574, 20.0695, 20.0451, 20.0974,
  20.0285, 20.0951, 20.0251, 20.0989, 20.0638, 20.0633, 20.006, 20.0375}

Mittelwert: 20.0573
```

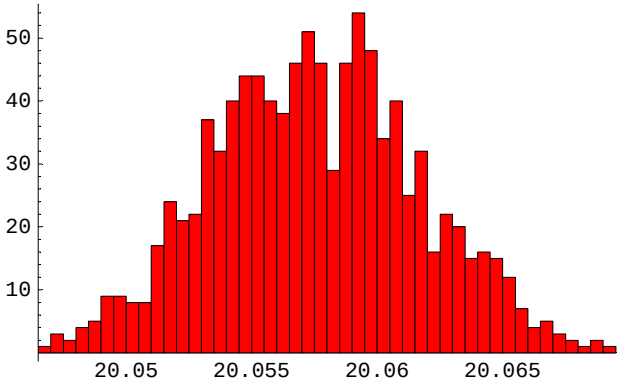



```
Kontrolle Anzahl:  False
```

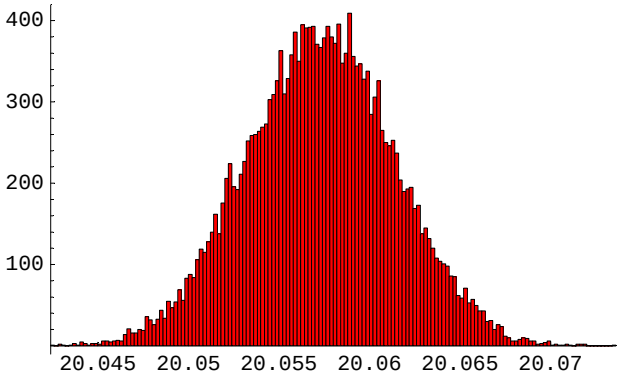
```
1000 mal bootstrapen
```



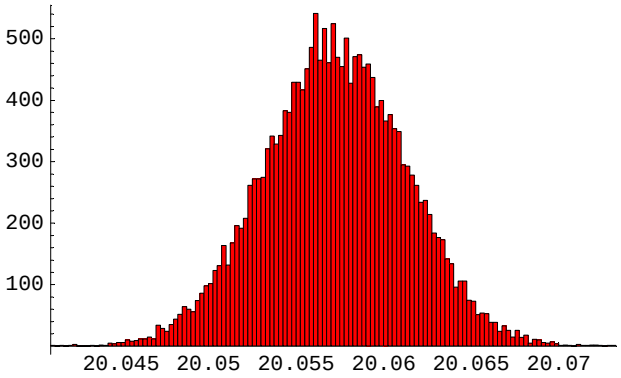

```
Nochmals 1000 mal bootstrapen
```




```
20000 mal bootstrapen
```




```
Nochmals 20000 mal bootstrapen
```



1000 Bootstrap-Kopien der Mittelwerte=

{20.0457, 20.0461, 20.0465, 20.0465, 20.0467, 20.0469, 20.0472, 20.0475, 20.0476, 20.0476,
20.0478, 20.048, 20.0485, 20.0486, 20.049, 20.0491, 20.0491, 20.0491, 20.0492, 20.0492,
20.0492, 20.0493, 20.0493, 20.0494, 20.0495, 20.0497, 20.0498, 20.0498, 20.0499, 20.0499,
20.0499, 20.05, 20.0501, 20.0501, 20.0501, 20.0502, 20.0502, 20.0502, 20.0503, 20.0503,
20.0503, 20.0503, 20.0504, 20.0504, 20.0504, 20.0505, 20.0506, 20.0507, 20.0507, 20.0508,
20.0508, 20.0508, 20.0509, 20.0509, 20.0509, 20.0509, 20.051, 20.051, 20.0511, 20.0511,
20.0511, 20.0512, 20.0512, 20.0513, 20.0513, 20.0513, 20.0514, 20.0514, 20.0514, 20.0514,
20.0515, 20.0515, 20.0517, 20.0517, 20.0517, 20.0518, 20.0518, 20.0518, 20.0518, 20.0518,
20.0518, 20.0519, 20.0519, 20.0519, 20.0519, 20.052, 20.052, 20.052, 20.0521, 20.0521,
20.0521, 20.0521, 20.0522, 20.0522, 20.0522, 20.0523, 20.0523, 20.0523, 20.0523, 20.0523,
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20.0525, 20.0525, 20.0525, 20.0526, 20.0526, 20.0526, 20.0526, 20.0526, 20.0526, 20.0526,
20.0526, 20.0527, 20.0527, 20.0527, 20.0527, 20.0528, 20.0528, 20.0528, 20.0528, 20.0528,
20.0528, 20.0528, 20.0529, 20.0529, 20.053, 20.053, 20.053, 20.053, 20.053, 20.0531,
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20.0535, 20.0535, 20.0535, 20.0536, 20.0536, 20.0536, 20.0536, 20.0536, 20.0536, 20.0536,
20.0536, 20.0537, 20.0537, 20.0537, 20.0537, 20.0538, 20.0538, 20.0538, 20.0539, 20.0539,
20.0539, 20.054, 20.054, 20.054, 20.054, 20.054, 20.054, 20.0541, 20.0541, 20.0541,
20.0541, 20.0541, 20.0541, 20.0541, 20.0541, 20.0542, 20.0542, 20.0542, 20.0542, 20.0542,
20.0542, 20.0542, 20.0543, 20.0543, 20.0543, 20.0543, 20.0543, 20.0543, 20.0543, 20.0543,
20.0543, 20.0544, 20.0544, 20.0544, 20.0544, 20.0544, 20.0544, 20.0544, 20.0544, 20.0545,
20.0545, 20.0545, 20.0545, 20.0546, 20.0546, 20.0546, 20.0546, 20.0547, 20.0547, 20.0547,
20.0547, 20.0547, 20.0547, 20.0548, 20.0548, 20.0548, 20.0548, 20.0548, 20.0548, 20.0548,
20.0548, 20.0548, 20.0548, 20.0549, 20.0549, 20.0549, 20.0549, 20.0549, 20.0549, 20.0549,
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20.0553, 20.0553, 20.0553, 20.0553, 20.0553, 20.0553, 20.0553, 20.0553, 20.0554, 20.0554,
20.0554, 20.0554, 20.0554, 20.0554, 20.0554, 20.0554, 20.0554, 20.0554, 20.0554, 20.0554,
20.0554, 20.0555, 20.0555, 20.0555, 20.0555, 20.0555, 20.0555, 20.0555, 20.0555, 20.0555,
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20.0557, 20.0557, 20.0557, 20.0557, 20.0558, 20.0558, 20.0558, 20.0558, 20.0558, 20.0558,
20.0558, 20.0558, 20.0558, 20.0558, 20.0558, 20.0558, 20.0559, 20.0559, 20.0559, 20.0559,
20.0559, 20.0559, 20.0559, 20.0559, 20.0559, 20.0559, 20.0559, 20.056, 20.056, 20.056,
20.056, 20.056, 20.056, 20.056, 20.056, 20.056, 20.056, 20.056, 20.056, 20.0561, 20.0561,
20.0561, 20.0562, 20.0562, 20.0562, 20.0562, 20.0562, 20.0562, 20.0563, 20.0563, 20.0563,
20.0563, 20.0563, 20.0563, 20.0563, 20.0563, 20.0563, 20.0563, 20.0563, 20.0563, 20.0563,
20.0563, 20.0563, 20.0564, 20.0564, 20.0564, 20.0564, 20.0564, 20.0564, 20.0564, 20.0564,
20.0565, 20.0565, 20.0565, 20.0565, 20.0565, 20.0565, 20.0565, 20.0565, 20.0565, 20.0565,
20.0565, 20.0566, 20.0566, 20.0566, 20.0566, 20.0566, 20.0566, 20.0566, 20.0566, 20.0566,
20.0566, 20.0566, 20.0567, 20.0567, 20.0567, 20.0567, 20.0567, 20.0567, 20.0567, 20.0567,
20.0568, 20.0568, 20.0568, 20.0568, 20.0568, 20.0568, 20.0568, 20.0568, 20.0568, 20.0568,
20.0568, 20.0568, 20.0568, 20.0568, 20.0569, 20.0569, 20.0569, 20.0569, 20.0569, 20.0569,
20.057, 20.057, 20.057, 20.057, 20.057, 20.057, 20.057, 20.0571, 20.0571, 20.0571,
20.0571, 20.0571, 20.0571, 20.0571, 20.0571, 20.0571, 20.0571, 20.0572, 20.0572, 20.0572,
20.0572, 20.0572, 20.0572, 20.0573, 20.0573, 20.0573, 20.0573, 20.0573, 20.0573, 20.0573,
20.0573, 20.0573, 20.0573, 20.0573, 20.0573, 20.0574, 20.0574, 20.0574, 20.0574, 20.0574,
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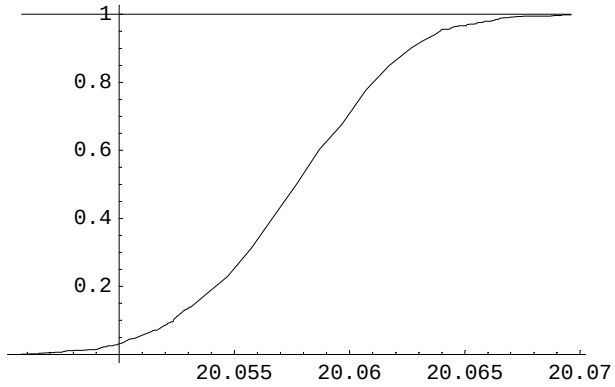
  20.0662, 20.0663, 20.0663, 20.0663, 20.0665, 20.0665, 20.0666, 20.0667, 20.0669,
  20.0669, 20.0671, 20.0673, 20.0675, 20.0688, 20.0689, 20.0692, 20.0692, 20.0696}

Mittelwert Min. bis Max.  = [20.0457, 20.0696]
```



Neue Messreihe

Work: Schrittweise Erzeugung einer Verteilungsfunktion, Fall 3 oder Beispiel 3

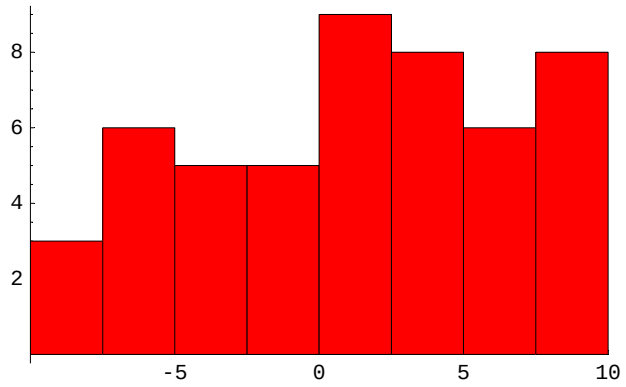
```
Remove["Global`*"]

<< Graphics`Graphics`;
<< Statistics`DataManipulation`;
```

Mit 80 Bootstrap-Kopien (Daten gewonnen wie eingangs)

```
M = Table[20 Random[] - 10, {n, 1, 50}];
Print["Mean " , Mean[M]]
Histogram[M];
u[j_] := Table[M[[Random[Integer, {1, 50}]]], {k, 1, 50}];
tWork2 = Table[Mean[u[j]], {j, 1, 80}];
tWork2
```

```
Mean 1.24228
```




```
{1.42903, 1.49431, 1.05637, 2.23933, 1.3913, 1.28952, 1.04789, 1.72, 1.52108,
 1.63265, 1.21034, 1.04011, 1.11942, 1.77399, 1.31788, -0.072453, 2.09519,
 0.604507, 0.554676, 2.06156, 0.539385, 1.59736, 3.0455, 0.974432, 2.20754,
 0.511723, 1.31987, 1.34912, 0.440375, 2.17679, 0.166767, 2.33985, 1.77789,
 1.17376, 0.825567, 2.36862, 0.999094, 0.961232, -0.598702, 0.874618, 1.35199,
 1.22433, 1.45268, 0.126996, 1.31303, 1.13748, 0.572546, 1.49724, 0.839246,
 1.00645, 0.527644, 2.24552, -0.766223, 0.382328, 0.904782, 0.894509,
 1.20621, 0.833766, 0.583961, 2.09603, 2.90094, 0.654102, 0.909385, 2.31257,
 2.35041, 1.28138, 1.85119, 1.87018, 1.621, 1.82182, 0.421995, 0.492341,
 2.33817, 0.693974, 0.792761, 3.26723, 0.524024, 1.82264, 3.14143, 1.42485}
```

```
h = Sort[tWork2]
```

```
{-0.766223, -0.598702, -0.072453, 0.126996, 0.166767, 0.382328, 0.421995,
 0.440375, 0.492341, 0.511723, 0.524024, 0.527644, 0.539385, 0.554676, 0.572546,
 0.583961, 0.604507, 0.654102, 0.693974, 0.792761, 0.825567, 0.833766, 0.839246,
 0.874618, 0.894509, 0.904782, 0.909385, 0.961232, 0.974432, 0.999094, 1.00645,
 1.04011, 1.04789, 1.05637, 1.11942, 1.13748, 1.17376, 1.20621, 1.21034,
 1.22433, 1.28138, 1.28952, 1.31303, 1.31788, 1.31987, 1.34912, 1.35199,
 1.3913, 1.42485, 1.42903, 1.45268, 1.49431, 1.49724, 1.52108, 1.59736,
 1.621, 1.63265, 1.72, 1.77399, 1.77789, 1.82182, 1.82264, 1.85119, 1.87018,
 2.06156, 2.09519, 2.09603, 2.17679, 2.20754, 2.23933, 2.24552, 2.31257,
 2.33817, 2.33985, 2.35041, 2.36862, 2.90094, 3.0455, 3.14143, 3.26723}
```

```
freq1 = Frequencies[tWork2];
freq2 = Table[{freq1[[k]][[2]], freq1[[k]][[1]]}, {k, 1, Length[freq1]}]
```

```
{{-0.766223, 1}, {-0.598702, 1}, {-0.072453, 1}, {0.126996, 1}, {0.166767, 1},
 {0.382328, 1}, {0.421995, 1}, {0.440375, 1}, {0.492341, 1}, {0.511723, 1},
 {0.524024, 1}, {0.527644, 1}, {0.539385, 1}, {0.554676, 1}, {0.572546, 1},
 {0.583961, 1}, {0.604507, 1}, {0.654102, 1}, {0.693974, 1}, {0.792761, 1},
 {0.825567, 1}, {0.833766, 1}, {0.839246, 1}, {0.874618, 1}, {0.894509, 1},
 {0.904782, 1}, {0.909385, 1}, {0.961232, 1}, {0.974432, 1}, {0.999094, 1},
 {1.00645, 1}, {1.04011, 1}, {1.04789, 1}, {1.05637, 1}, {1.11942, 1},
 {1.13748, 1}, {1.17376, 1}, {1.20621, 1}, {1.21034, 1}, {1.22433, 1},
 {1.28138, 1}, {1.28952, 1}, {1.31303, 1}, {1.31788, 1}, {1.31987, 1},
 {1.34912, 1}, {1.35199, 1}, {1.3913, 1}, {1.42485, 1}, {1.42903, 1}, {1.45268, 1},
 {1.49431, 1}, {1.49724, 1}, {1.52108, 1}, {1.59736, 1}, {1.621, 1}, {1.63265, 1},
 {1.72, 1}, {1.77399, 1}, {1.77789, 1}, {1.82182, 1}, {1.82264, 1}, {1.85119, 1},
 {1.87018, 1}, {2.06156, 1}, {2.09519, 1}, {2.09603, 1}, {2.17679, 1}, {2.20754, 1},
 {2.23933, 1}, {2.24552, 1}, {2.31257, 1}, {2.33817, 1}, {2.33985, 1},
 {2.35041, 1}, {2.36862, 1}, {2.90094, 1}, {3.0455, 1}, {3.14143, 1}, {3.26723, 1}}
```

```
freq2 = Table[{freq1[[k, 2]], freq1[[k, 1]]}, {k, 1, Length[freq1]}]
```

```
{{-0.766223, 1}, {-0.598702, 1}, {-0.072453, 1}, {0.126996, 1}, {0.166767, 1},
 {0.382328, 1}, {0.421995, 1}, {0.440375, 1}, {0.492341, 1}, {0.511723, 1},
 {0.524024, 1}, {0.527644, 1}, {0.539385, 1}, {0.554676, 1}, {0.572546, 1},
 {0.583961, 1}, {0.604507, 1}, {0.654102, 1}, {0.693974, 1}, {0.792761, 1},
 {0.825567, 1}, {0.833766, 1}, {0.839246, 1}, {0.874618, 1}, {0.894509, 1},
 {0.904782, 1}, {0.909385, 1}, {0.961232, 1}, {0.974432, 1}, {0.999094, 1},
 {1.00645, 1}, {1.04011, 1}, {1.04789, 1}, {1.05637, 1}, {1.11942, 1},
 {1.13748, 1}, {1.17376, 1}, {1.20621, 1}, {1.21034, 1}, {1.22433, 1},
 {1.28138, 1}, {1.28952, 1}, {1.31303, 1}, {1.31788, 1}, {1.31987, 1},
 {1.34912, 1}, {1.35199, 1}, {1.3913, 1}, {1.42485, 1}, {1.42903, 1}, {1.45268, 1},
 {1.49431, 1}, {1.49724, 1}, {1.52108, 1}, {1.59736, 1}, {1.621, 1}, {1.63265, 1},
 {1.72, 1}, {1.77399, 1}, {1.77789, 1}, {1.82182, 1}, {1.82264, 1}, {1.85119, 1},
 {1.87018, 1}, {2.06156, 1}, {2.09519, 1}, {2.09603, 1}, {2.17679, 1}, {2.20754, 1},
 {2.23933, 1}, {2.24552, 1}, {2.31257, 1}, {2.33817, 1}, {2.33985, 1},
 {2.35041, 1}, {2.36862, 1}, {2.90094, 1}, {3.0455, 1}, {3.14143, 1}, {3.26723, 1}}
```

```
Length[freq1]
```

```
80
```

```
freq3 = Table[freq1[[k, 2]], {k, 1, Length[freq1]}]
```

```
{-0.766223, -0.598702, -0.072453, 0.126996, 0.166767, 0.382328, 0.421995,
 0.440375, 0.492341, 0.511723, 0.524024, 0.527644, 0.539385, 0.554676, 0.572546,
 0.583961, 0.604507, 0.654102, 0.693974, 0.792761, 0.825567, 0.833766, 0.839246,
 0.874618, 0.894509, 0.904782, 0.909385, 0.961232, 0.974432, 0.999094, 1.00645,
 1.04011, 1.04789, 1.05637, 1.11942, 1.13748, 1.17376, 1.20621, 1.21034,
 1.22433, 1.28138, 1.28952, 1.31303, 1.31788, 1.31987, 1.34912, 1.35199,
 1.3913, 1.42485, 1.42903, 1.45268, 1.49431, 1.49724, 1.52108, 1.59736,
 1.621, 1.63265, 1.72, 1.77399, 1.77789, 1.82182, 1.82264, 1.85119, 1.87018,
 2.06156, 2.09519, 2.09603, 2.17679, 2.20754, 2.23933, 2.24552, 2.31257,
 2.33817, 2.33985, 2.35041, 2.36862, 2.90094, 3.0455, 3.14143, 3.26723}
```

```
Select[freq3, # ≤ 1 &]
```

```
{-0.766223, -0.598702, -0.072453, 0.126996, 0.166767, 0.382328, 0.421995,
 0.440375, 0.492341, 0.511723, 0.524024, 0.527644, 0.539385, 0.554676, 0.572546,
 0.583961, 0.604507, 0.654102, 0.693974, 0.792761, 0.825567, 0.833766, 0.839246,
 0.874618, 0.894509, 0.904782, 0.909385, 0.961232, 0.974432, 0.999094}
```

```
Length[Select[freq3, # ≤ 1 &]]
```

```
30
```

```
Take[freq2, Length[Select[freq3, # ≤ 1 &]]]
```

```
{{-0.766223, 1}, {-0.598702, 1}, {-0.072453, 1}, {0.126996, 1}, {0.166767, 1},
 {0.382328, 1}, {0.421995, 1}, {0.440375, 1}, {0.492341, 1}, {0.511723, 1},
 {0.524024, 1}, {0.527644, 1}, {0.539385, 1}, {0.554676, 1}, {0.572546, 1},
 {0.583961, 1}, {0.604507, 1}, {0.654102, 1}, {0.693974, 1}, {0.792761, 1},
 {0.825567, 1}, {0.833766, 1}, {0.839246, 1}, {0.874618, 1}, {0.894509, 1},
 {0.904782, 1}, {0.909385, 1}, {0.961232, 1}, {0.974432, 1}, {0.999094, 1}}
```

```
tabX = Take[freq2, Length[Select[Table[freq1[[k, 2]], {k, 1, Length[freq1]}], # ≤ 0 &]]]
```

```
{{-0.766223, 1}, {-0.598702, 1}, {-0.072453, 1}}
```

```
tabX2 = Table[tabX[[k]][[2]], {k, 1, Length[tabX]} ]
```

```
{1, 1, 1}
```

```
Apply[Plus, tabX2]
```

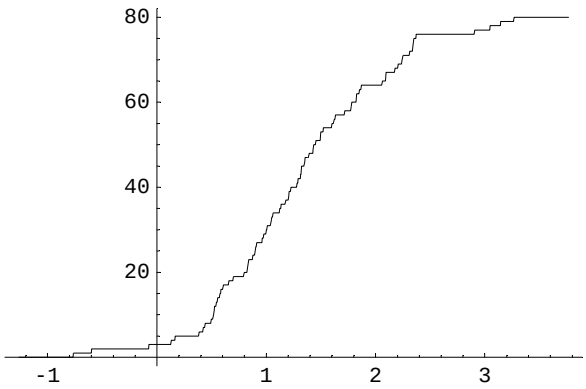
```
3
```

```
freq1 = Frequencies[tWork2];
f[x_] := Apply[Plus,
  Table[Take[freq2, Length[Select[Table[freq1[[k, 2]], {k, 1, Length[freq1]}],
        # ≤ x &]]][[k]][[2]], {k, 1, Length[Take[freq2,
      Length[Select[Table[freq1[[k, 2]], {k, 1, Length[freq1]}], # ≤ x &]]]]} ]]
```

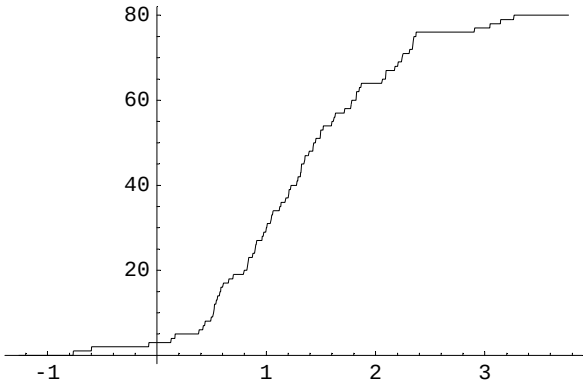
```
{f[0], f[0.5], f[1], f[2], f[3]}
```

```
{3, 9, 30, 64, 77}
```

```
Plot[f[x], {x, Min[h] - 0.5, Max[h] + 0.5}];
```



```
freq1 = Frequencies[tWork2];
F1[x_] := Apply[Plus,
   Table[Take[freq1, Length[Select[Table[freq1[[k1, 2]], {k1, 1, Length[freq1]}],
         # ≤ x &]]][[k1]][[1]], {k1, 1, Length[Take[freq 1,
       Length[Select[Table[freq1[[k, 2]], {k, 1, Length[freq1]}], # ≤ x &]]]]} ]];
Plot[F1[x], {x, Min[h] - 0.5, Max[h] + 0.5}];
```



Mit 1000 Bootstrap-Kopien

```
ttt = Sort[Table[Mean[u[j]], {j, 1, 1000}]];
freq = Union[Frequencies[ttt]];
F[x_] := Apply[Plus, Table[Take[freq, Length[Select[Table[
          freq[[k1, 2]], {k1, 1, Length[freq]}], # ≤ x &]]][[k1]][[1]],
    {k1, 1, Length[Take[freq, Length[Select[Table[freq[[k, 2]],
          {k, 1, Length[freq]}], # ≤ x &]]]]} ]]  / Length[ttt] ;
Plot[{F[x], 1}, {x, Min[h] - 0.5, Max[h] + 0.5}];
```