Kurzlösungen Analysis 02

Falls eine Aufgabe von einer Mehrheit nicht verstanden wird, so soll ie Aufgabe in der letzten Analysis-Lektion der Woche thematisiert werden.

Falls eine Aufgabe von einer Meinderheit nicht verstanden wird, so soll man dafür möglichst rasch für eine Sprechstunde anmelden.

Falls eine Aufgabe sowieso klar ist, bedeutet ein weiterer Zeitaufwand nur einen Zeitverlust. Falls niemand Verständnisprobleme vorbringt, so kann angenommen werden, dass die Aufgabe erledigt ist.

Uebung 1

a

Quadrieren, dann siehr man die Ungleichheitsbeziehung (für nicht negative Werte ändert quadrieren die Ungleichheitsbeziehung nicht)

$$(\text{Abs}[x] - \text{Abs}[y])^2 = \text{Abs}[x]^2 - 2\,\text{Abs}[x]\,\text{Abs}[y] + \text{Abs}[y]^2$$
$$\leq x^2 - 2xy + y^2 = (x-y)^2$$
$$\leq \text{Abs}[x]^2 + 2\,\text{Abs}[x]\,\text{Abs}[y] + \text{Abs}[y]^2 = (\text{Abs}[x] + \text{Abs}[y])^2$$

$$(\text{Abs}[x] - \text{Abs}[y])^2 = \text{Abs}[x]^2 - 2\,\text{Abs}[x]\,\text{Abs}[y] + \text{Abs}[y]^2$$
$$\leq x^2 + 2xy + y^2 = (x+y)^2$$
$$\leq \text{Abs}[x]^2 + 2\,\text{Abs}[x]\,\text{Abs}[y] + \text{Abs}[y]^2 = (\text{Abs}[x] + \text{Abs}[y])^2$$

b

Out[3]= $2^{-6+\sqrt{2}}$

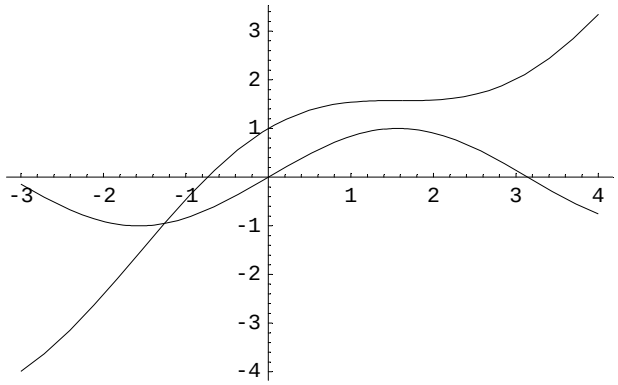
Out[4]= 0.0416429

c

Transzendente Gleichung nuimerisch oder graphisch lösen

$$\{x \to -1.25873\}$$

```
Plot[{Sin[x],x+Cos[x]},{x,-3,4}];
```



---

d

sin(2x-4) = 0 ===> x = 2 + n /2

(5 Cos[3x+2]+7 Sin[3x+2]-1) = 0
Löse (5 Cos[3x+2]+7 Sin[3x+2]-1)/Sqrt[5^2+7^2] = 0, denn es ist
ArcCos[7 /$\sqrt{74}$]=ArcSin[5 /$\sqrt{74}$] (weil gilt Sin[ ]^2 + Cos[ ]^2 = 1). Damit gilt
5 Cos[3x+2] + 7 Sin[3x+2] -1)/Sqrt[5^2+7^2]
= 5/$\sqrt{74}$ Cos[3x+2] + 7 /$\sqrt{74}$ Sin[3x+2] - 1/$\sqrt{74}$
= Sin[ArcSin[5/$\sqrt{74}$]]Cos[3x+2] + Cos[ArcCos[7 /$\sqrt{74}$]] Sin[3x+2] - 1/$\sqrt{74}$
= Sin[ArcSin[5/$\sqrt{74}$]+3x+2] - 1/$\sqrt{74}$ =0. Damit ist ArcSin[5/$\sqrt{74}$]+3x+2= ArcSin[1/$\sqrt{74}$].
Hier ist x linear berechenbar.

Sqrt[5^2+7^2] = $\sqrt{74}$

Maschinenlösung

$$\left\{x \to \frac{1}{3}\left(-2+\text{ArcSin}\left[\frac{1}{\sqrt{74}}\right]-\text{ArcSin}\left[\frac{5}{\sqrt{74}}\right]\right)\right\}$$

Numerisch:

```
{{x → -0.834579}}
```

Weiter:

```
{{x → -0.834579}, {x → -2.92897}, {x → 1.25982},
 {x → 0.134944}, {x → 2.22934}, {x → -1.95945}}
```

e

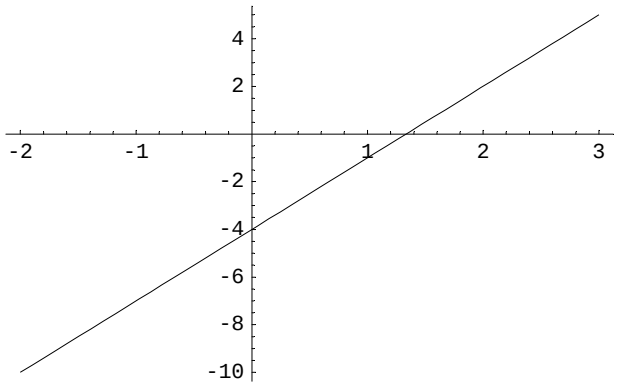
Idee: Schreibe alle Basen als e hoch Logarithömus naturalis von der Basis! Dann findet öman eine Gleichung für die Exponenten!

$$\{x \to 1\}$$

Uebung 2

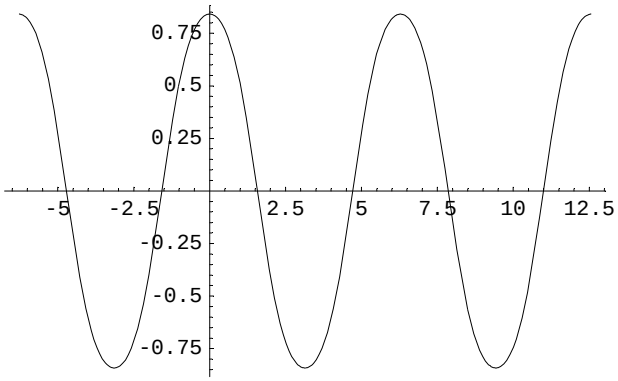
a

3x-4



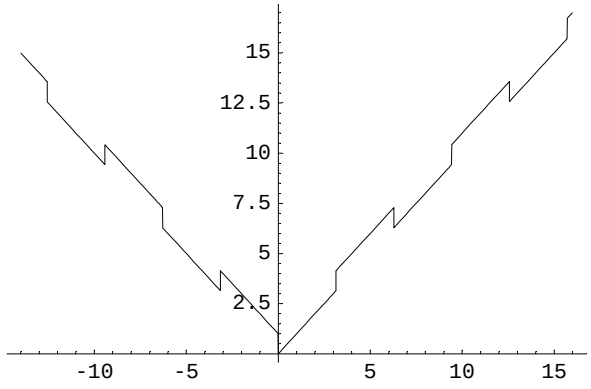
b

Sin[Cos[x]



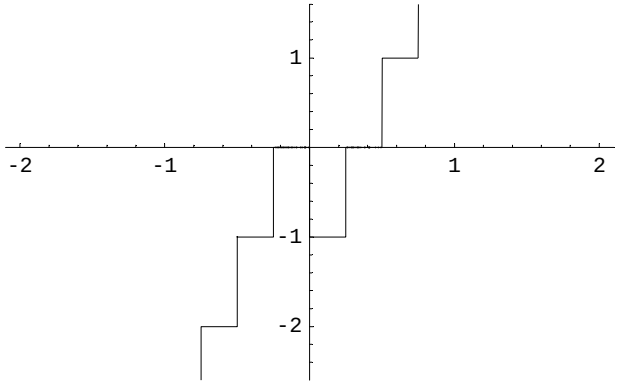
c

Abs[x]-Floor[Sin[x]]



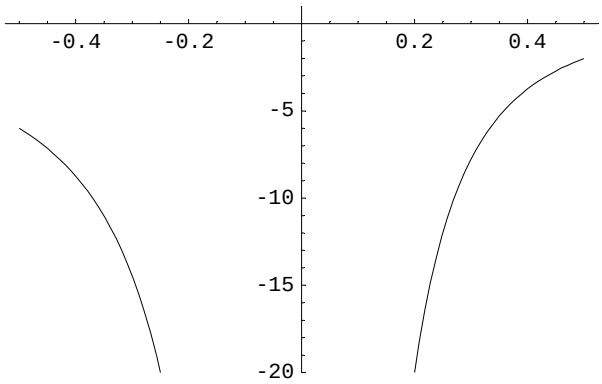
d

Floor[4x]-Sign[x]



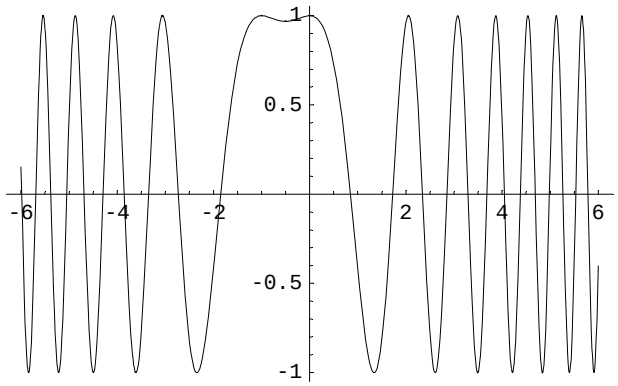
e

1/x-1/(x^2)



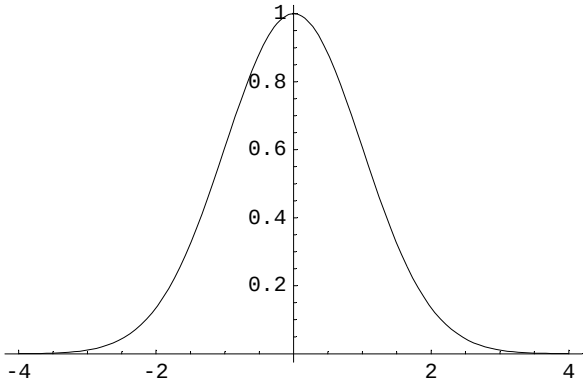
f

Cos[x^2+x]



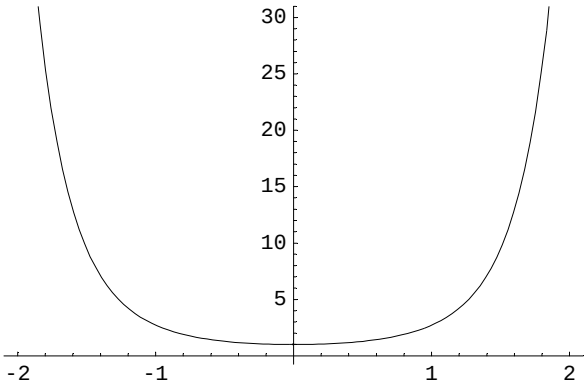
g

e^(-1/2 x^2)



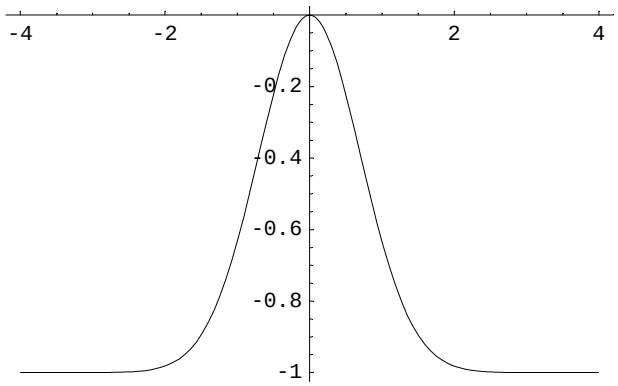
h

e^(x^2)



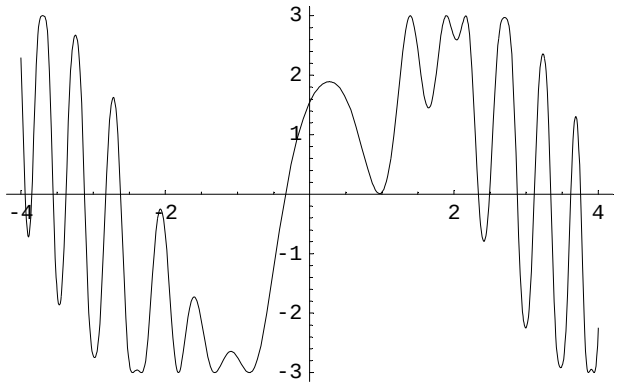
i

e^(- x^2)-1



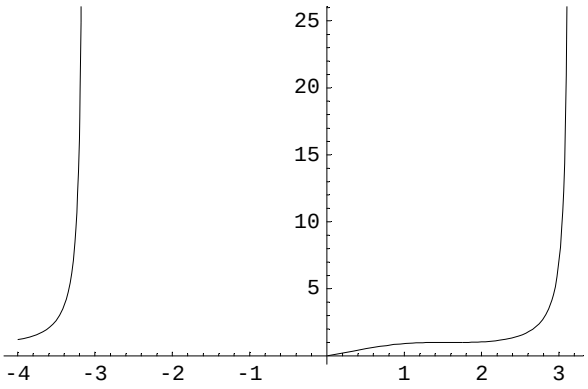
j

3 Sin[Cos[2x^2+1]



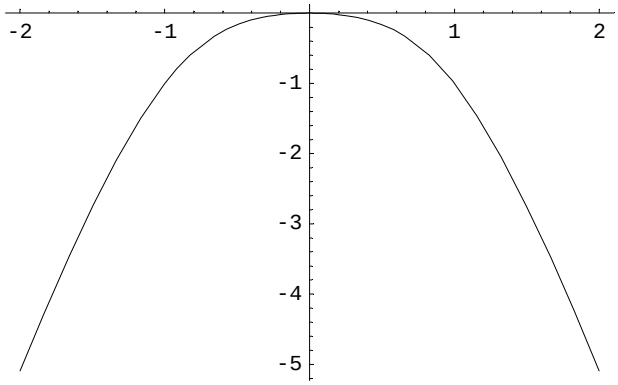
k

Sin[x]^Cos[x] nicht immer definiert, wenn der Sinus negativ wird....



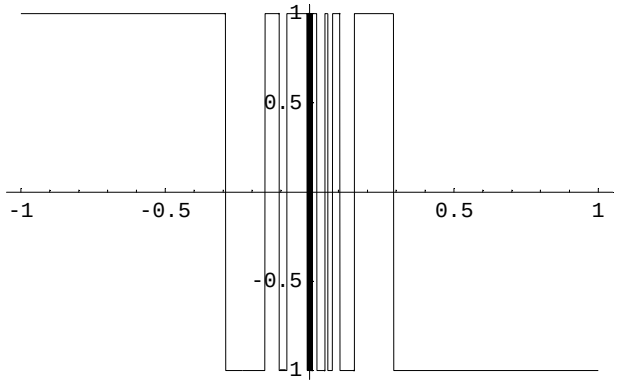
l

Log[(x^2+2)/(x^4+2)



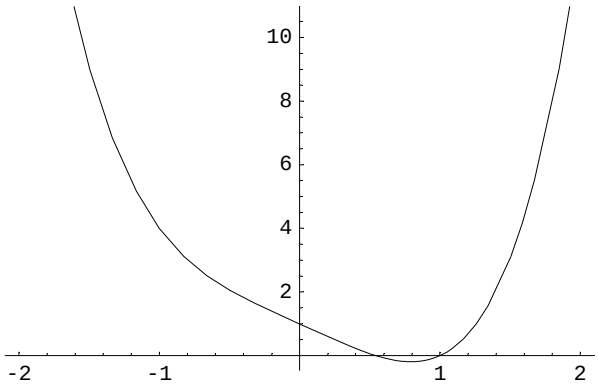
m

Sign[x^2 Sin[x-1/x]]



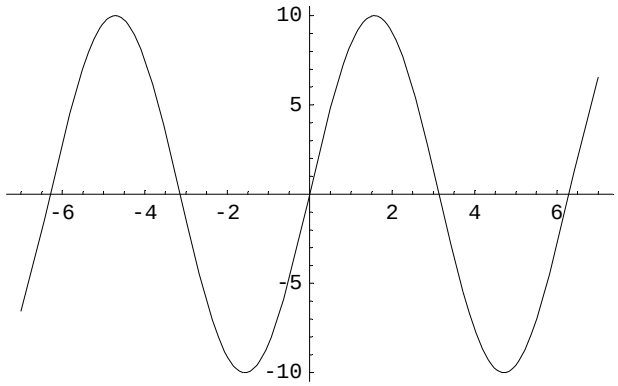
n

x^4-2x+1

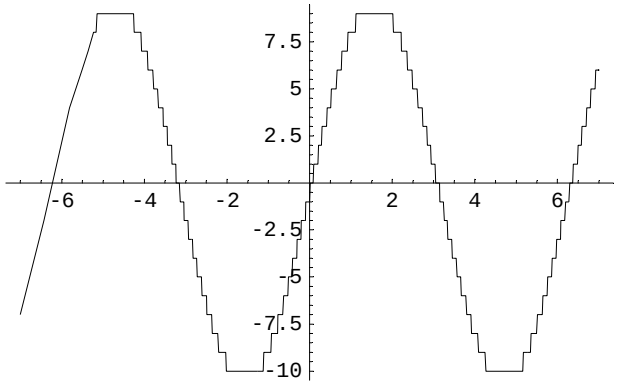


o

10 Sin[x]

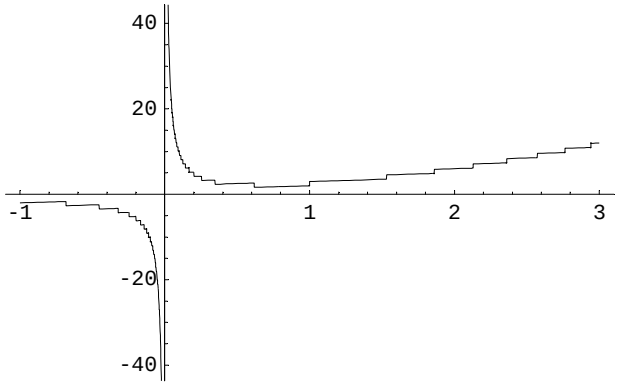


Floor[10 Sin[x]]

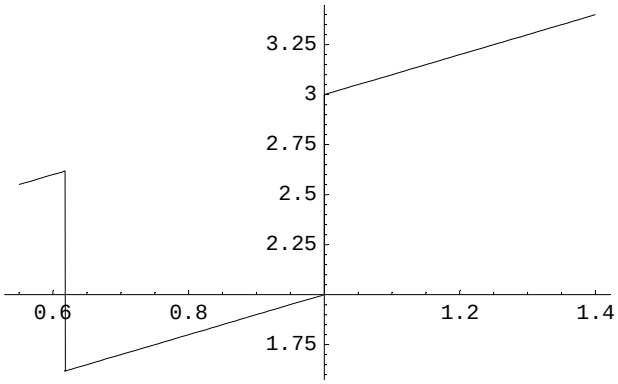


p

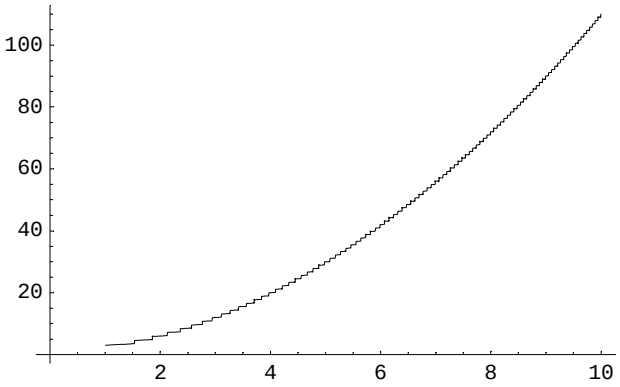
x+Floor[1/x+x^2]



x+Floor[1/x+x^2],{x,0.55,1.4}

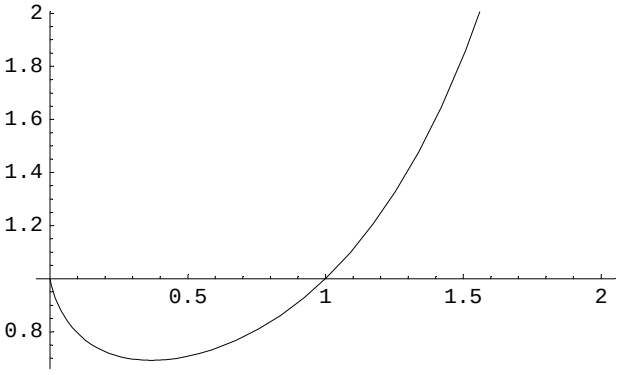


x+Floor[1/x+x^2]

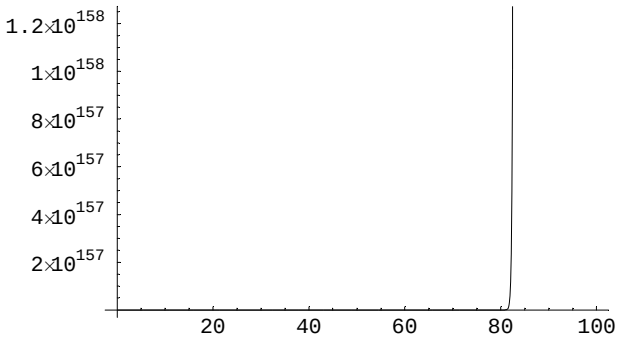


q

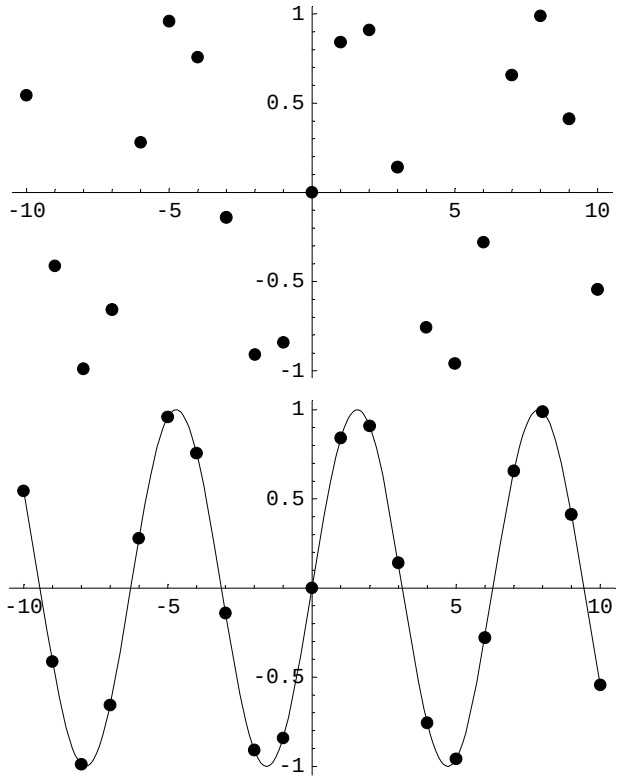
x^x



x^x

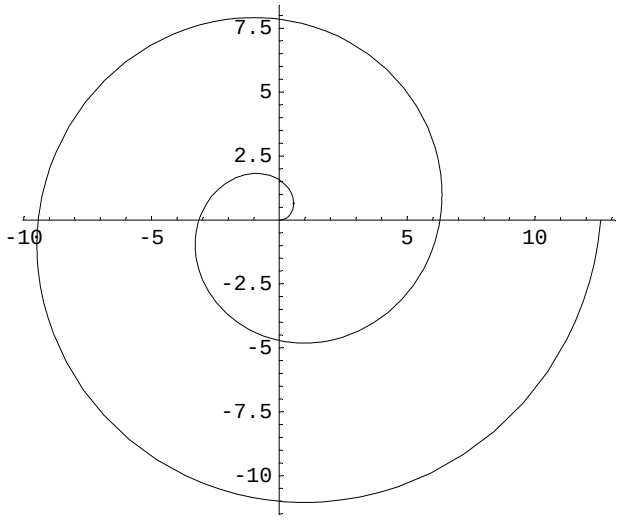


Uebung 3 L



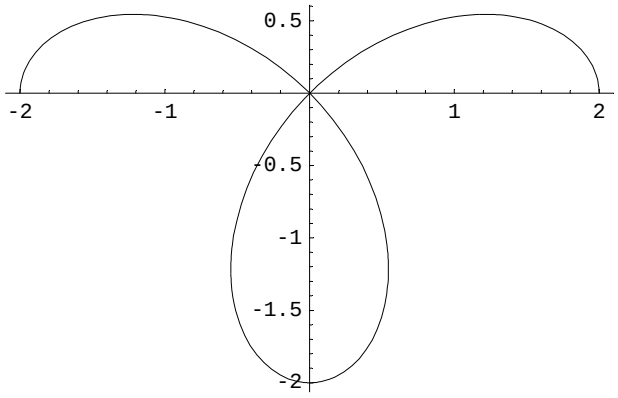
Uebung 4 L

polarPlot[t, {t,0,4Pi}];



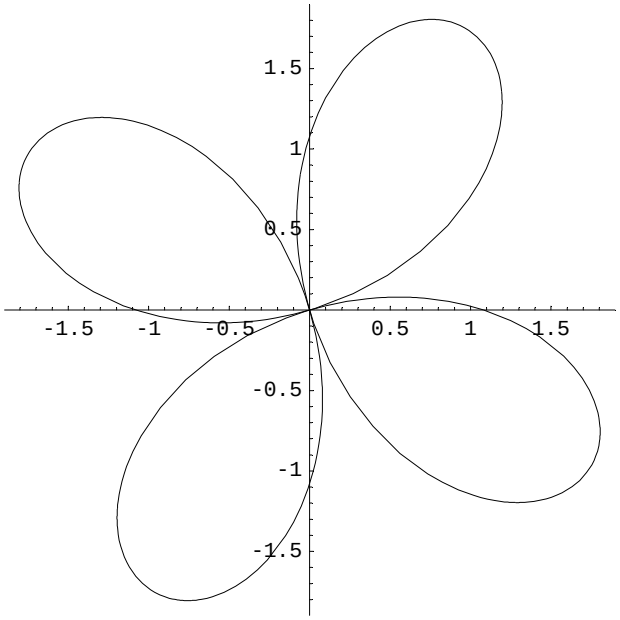
a

2 Cos[2 t]



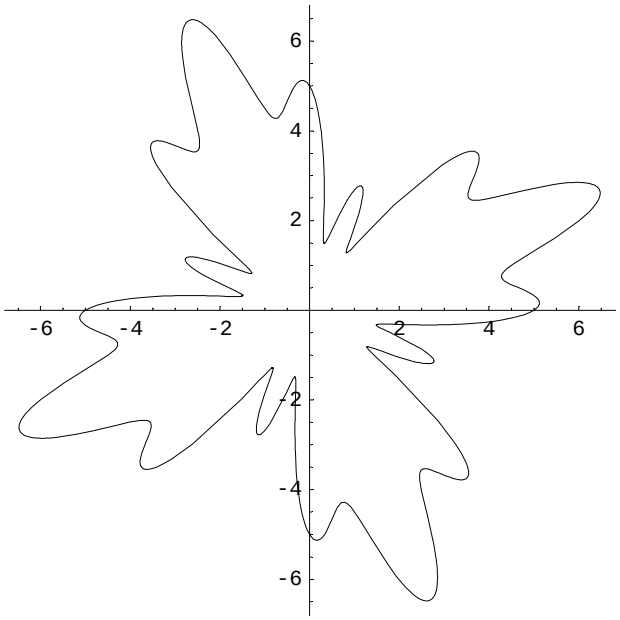
b

2 Cos[2 t+1]



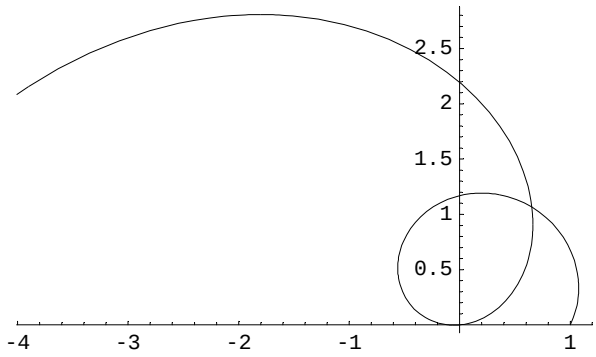
c

4+2Sin[4t]+Cos[16t]



d

1+t/2-t^2 /4

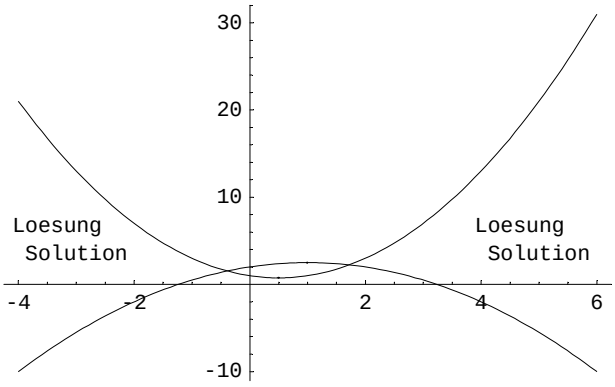


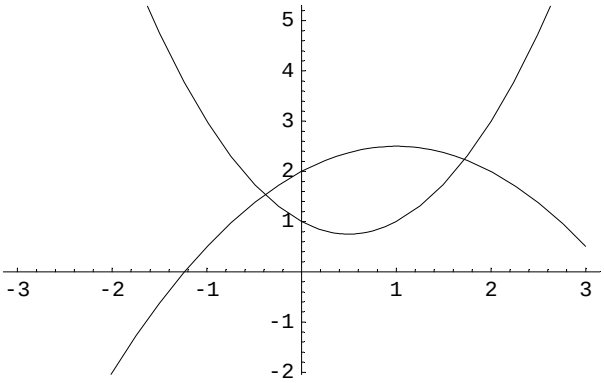
Uebung 5

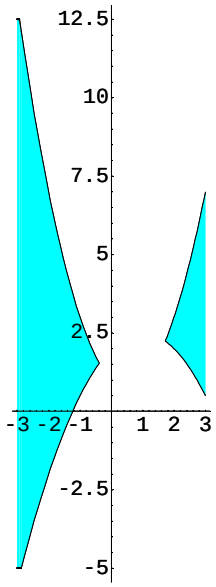
Schnittpunkte der Kurven:

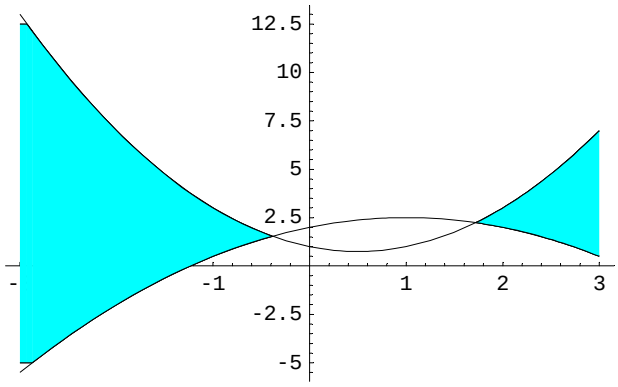
$$\left\{x \to \frac{1}{3}\left(2-\sqrt{10}\right),\ x \to \frac{1}{3}\left(2+\sqrt{10}\right)\right\}$$

$$\{x \to -0.387426,\ x \to 1.72076\}$$









Ueb/Ex 6

Quadratische Ergänzung, in der Lektion gemacht, Siehe auch Skript Analysis.

Uebung 7

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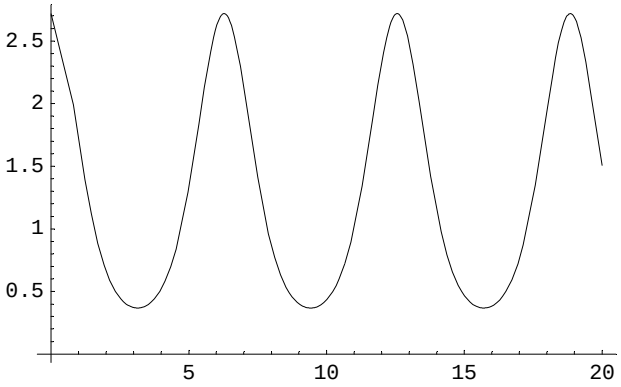
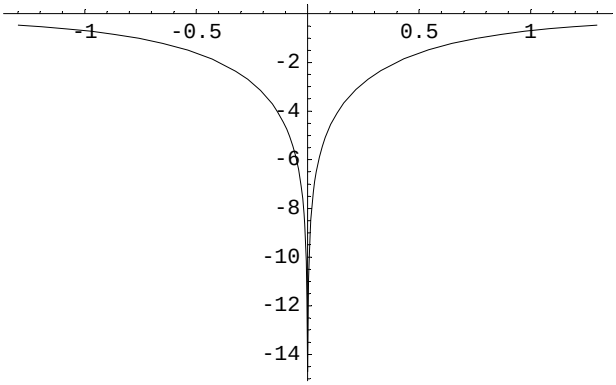
a

e^Cos[x] beschränkt, stückweise monoton



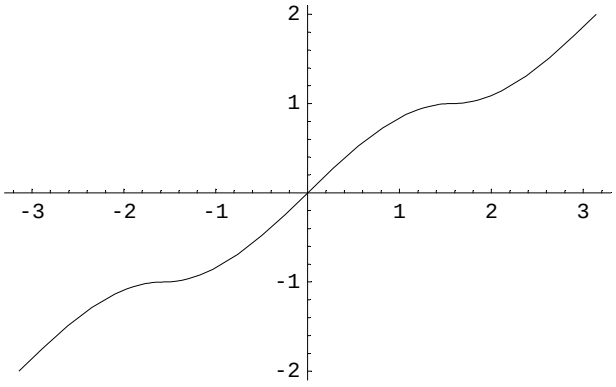
b

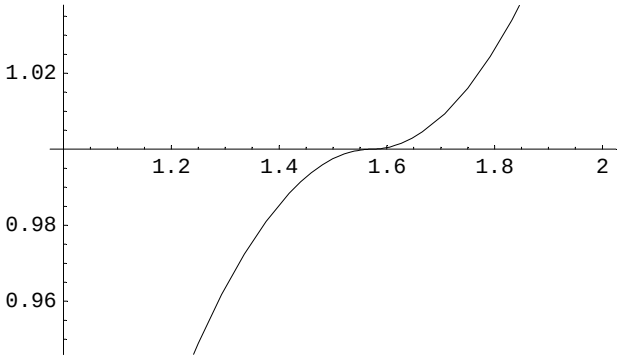
Log[x^2/(x^2+1)] stückweise monoton, nach oben beschränkt



c

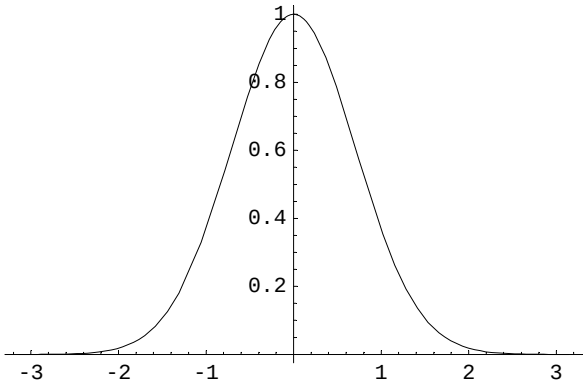
Funktion monoton wachsend





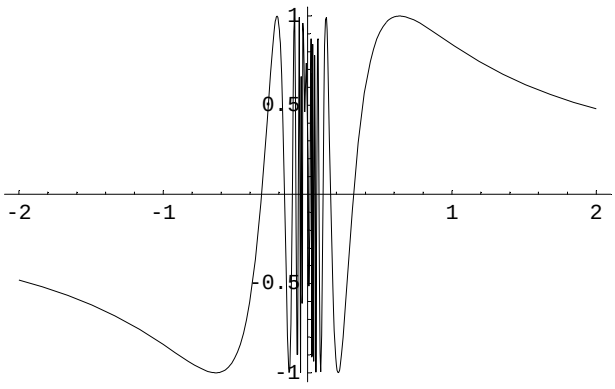
d

e^(-x^2) beschränkt, stückweise monoton



Uebung 8

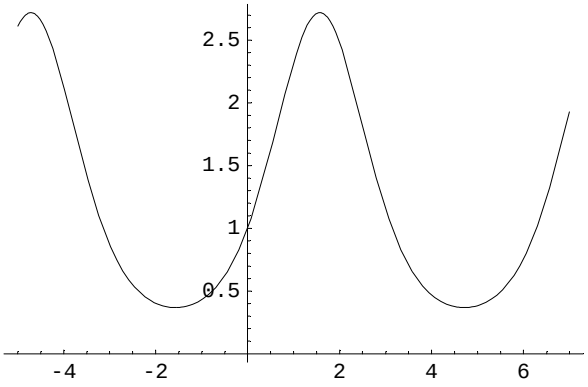
a



Sin[1/x] :Beschränkt, in 0: nicht definiert

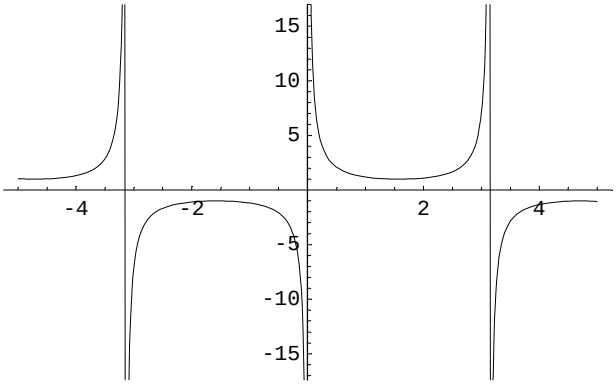
b

Plot[E^Sin[x], {x,-5,7}];



e^Sin[x]: Beschränkt, periodisch

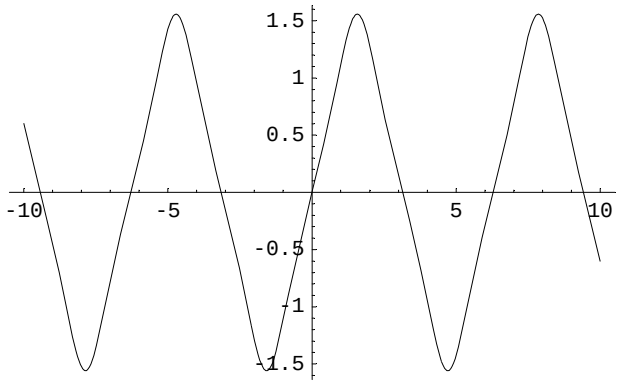
c



1/Sin[x]: Polstellen

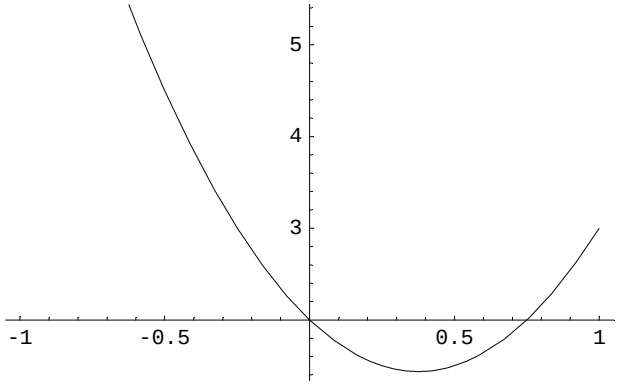
d

```
Plot[Tan[Sin[x]], {x, -10, 10}];
```



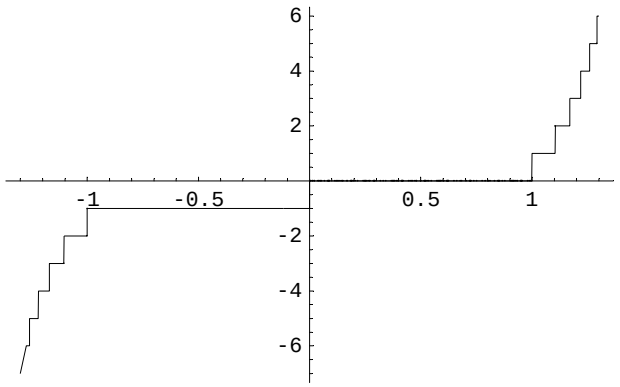
Tan[Sin[x]]: Beschränkt, periodisch

e



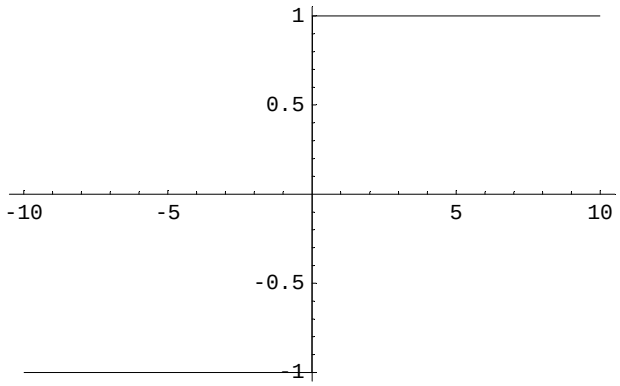
4x^2-3x+2: Beschränkt

f



Floor[x^7]: Sprüng

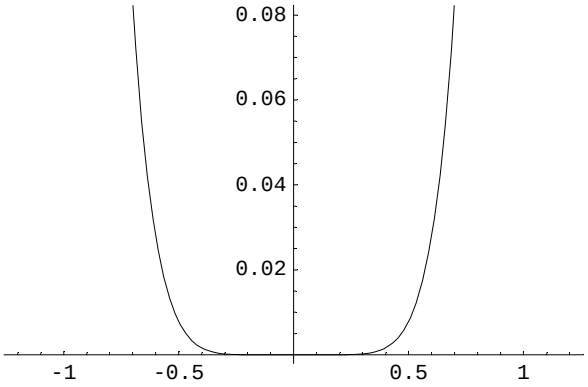
g



Sign[x^7]: Beschränkt, monoton wachsend, Sprünge bei 0

h

```
Plot[x^7 Sign[x^7], {x, -1.2, 1.2}];
```

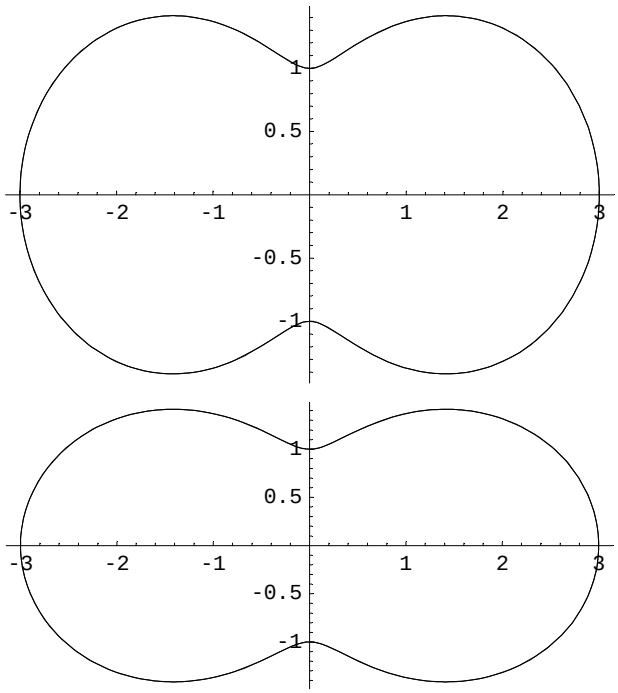


x^7 Sign[x^7]: Stückweise wtreng monoton

Uebung 8

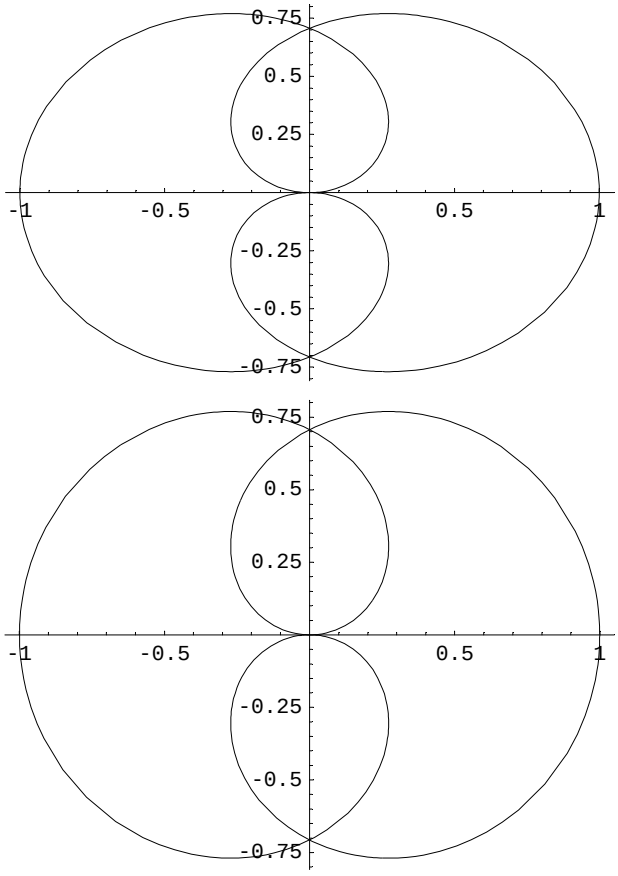
a

Cos[2fi]+2


1
0.5
-3
-2
-1
1
2
3
-0.5
-1
1
0.5
-3
-2
-1
1
2
3
-0.5
-1

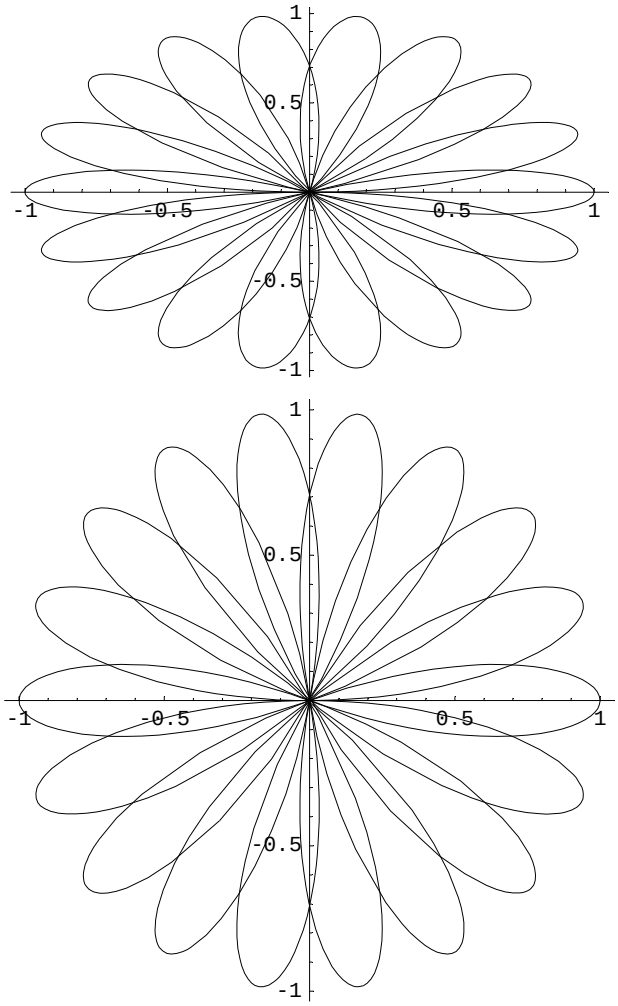
b

Cos[fi/2]


0.75
0.5
0.25
-1
-0.5
0.5
1
-0.25
-0.5
-0.75
0.75
0.5
0.25
-1
-0.5
0.5
1
-0.25
-0.5
-0.75

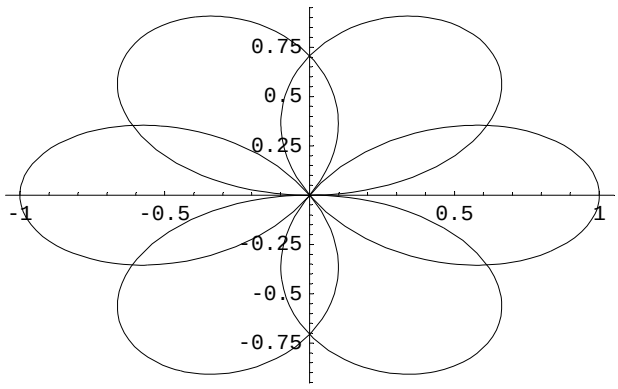
c

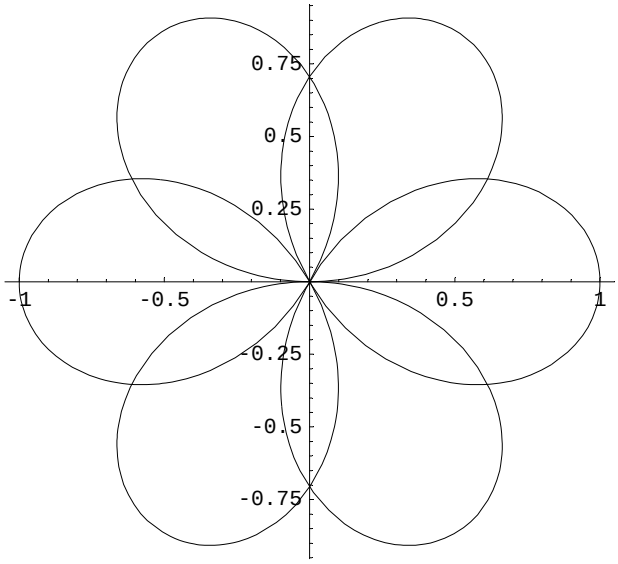
Cos[4 fi+fi/2]



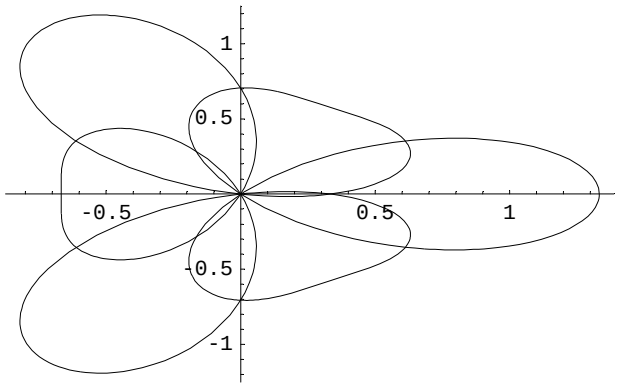
d

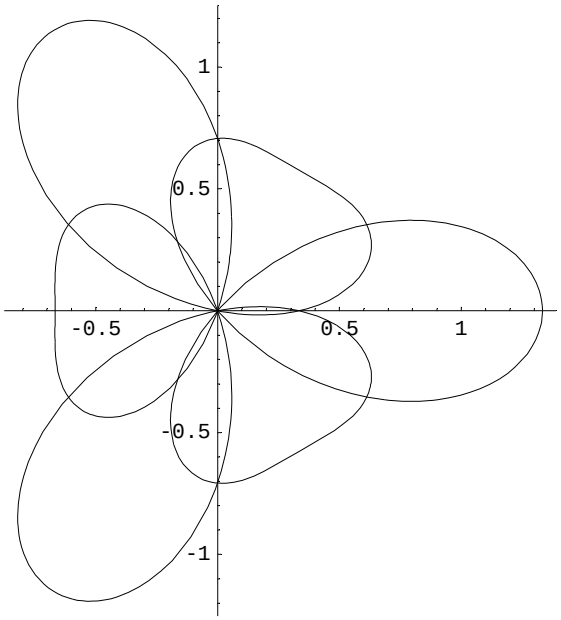
Cos[fi+fi/2]



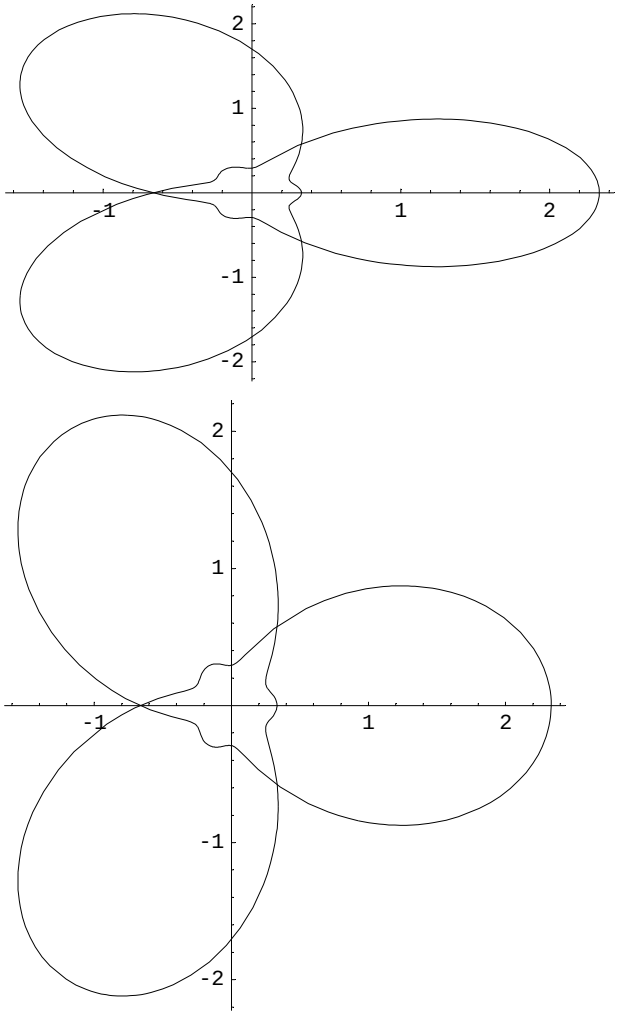


Cos[fi+fi/2]+1/3Cos[3fi]

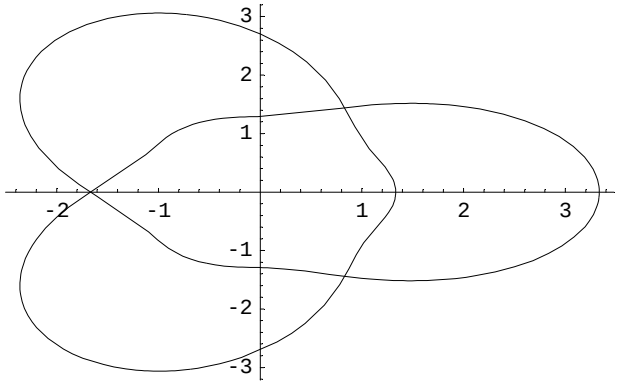


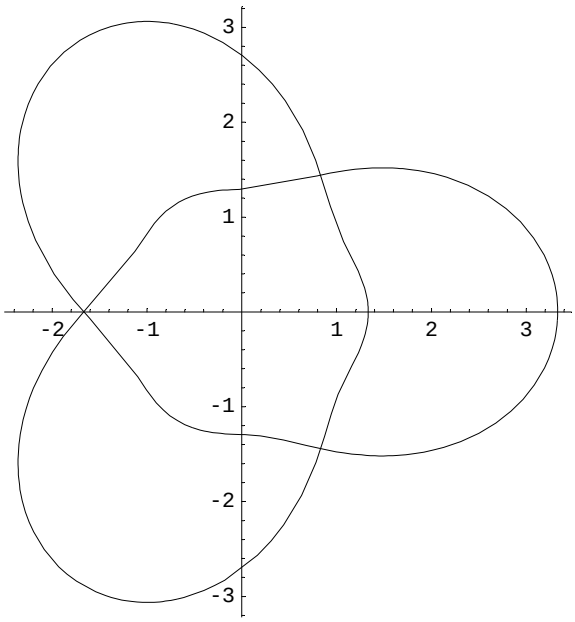


Cos[fi+fi/2]+1/3Cos[3fi]+1



Cos[fi+fi/2]+1/3Cos[3fi]+2




3
2
1
-2
-1
1
2
3
-1
-2
-3