Lösungen

1

```
Remove["Global`*"]
```

■ a

```
f1[x_] := (x - 1) (x + 1) (x^2 - 1) + 3 x^3 + 6 x^2 + 3 x // Expand; f1[x]
```

$1 + 3x + 4x^2 + 3x^3 + x^4$

```
f1'[x]
```

$3 + 8x + 9x^2 + 4x^3$

■ b

```
f2[x_] := Sin[E^x - 1] Sinh[x]; f2[x]
```

$-\mathrm{Sin}[1 - e^x]\ \mathrm{Sinh}[x]$

```
f2'[x]
```

$-\mathrm{Cosh}[x]\ \mathrm{Sin}[1 - e^x] + e^x\ \mathrm{Cos}[1 - e^x]\ \mathrm{Sinh}[x]$

■ c

```
f3[x_] := (x + 1) (x + 3) / (x^2 - 1); f3[x]
```

$$\frac{(1 + x)(3 + x)}{-1 + x^2}$$

```
f3[x] // Simplify
```

$$\frac{3 + x}{-1 + x}$$

```
f3[x] // Expand
```

$$\frac{3}{-1 + x^2} + \frac{4x}{-1 + x^2} + \frac{x^2}{-1 + x^2}$$

```
f3'[x]
```

$$-\frac{2x(1 + x)(3 + x)}{(-1 + x^2)^2} + \frac{1 + x}{-1 + x^2} + \frac{3 + x}{-1 + x^2}$$

```
f3'[x] // Simplify
```

$$-\frac{4}{(-1 + x)^2}$$

■ d

```
f4[x_] := (3 x)^(2 x); f4[x]
```

$3^{2x} x^{2x}$

```
f4'[x]
```

$2 \times 3^{2x} x^{2x}\ \mathrm{Log}[3] + 3^{2x} x^{2x} (2 + 2\ \mathrm{Log}[x])$

```
f4'[x] // Simplify
```

$2 \times 9^x x^{2x} (1 + \text{Log}[3] + \text{Log}[x])$

```
% // N
```

$2. \times 9.^x x^{2. x} (2.09861 + \text{Log}[x])$

- **e**

```
4 (x^3 + 1) / (5 (x + 1)) - (E^(x + 1) - 1) / (x^2 - 1) // TraditionalForm
```

$$\frac{4(x^3+1)}{5(x+1)} - \frac{e^{x+1}-1}{x^2-1}$$

```
Limit[4 (x^3 + 1) / (5 (x + 1)) - (E^(x + 1) - 1) / (x^2 - 1), x → -1]
```

$$\frac{29}{10}$$

2

```
Remove["Global`*"]
```

- **a**

```
f5[x_] := x Sin[x];
Tan[35 °] // N
```

0.700208

```
x0 = x /. FindRoot[Evaluate[f5'[t] - Tan[35 Degree] == 0] /. t → x, {x, 1}]
```

0.366326

```
y0 = f5[x0]
```

0.131213

```
x1 = x /. FindRoot[Evaluate[f5'[t] - Tan[35 Degree] == 0] /. t → x, {x, 3}]
```

1.73634

```
y1 = f5[x1]
```

1.7126

```
x0 = x /. FindRoot[Evaluate[f5'[t] - Tan[35 Degree] == 0] /. t → x, {x, 100}]
```

98.9773

```
x0 = x /. FindRoot[Evaluate[f5'[t] - Tan[35 Degree] == 0] /. t → x, {x, 80}]
```

80.1318

- **Es existieren weitere (beliebig viele) Lösungen, denn der Sinus ist periodisch. Beispiele:**

```
Join[{{"Nummer q ", "Wert xq "}},
  Table[{q, xq = x /. FindRoot[Evaluate[f5'[t] - Tan[35 Degree] == 0] /. t → x, {x, q}]},
   {q, 0.5, 100, 1}]] // TableForm
```

Nummer q	Wert xq
0.5	0.366326
1.5	1.73634
2.5	1.73634
3.5	-2.27151
4.5	5.04466
5.5	5.04466
6.5	-4.77481

7.5	7.89189
8.5	7.89189
9.5	1.73634
10.5	11.1476
11.5	11.1476
12.5	7.89189
13.5	14.1583
14.5	14.1583
15.5	7.89189
16.5	17.3765
17.5	17.3765
18.5	17.3765
19.5	20.435
20.5	20.435
21.5	20.435
22.5	23.6338
23.5	23.6338
24.5	23.6338
25.5	26.7148
26.5	26.7148
27.5	26.7148
28.5	29.902
29.5	29.902
30.5	29.902
31.5	32.9958
32.5	32.9958
33.5	32.9958
34.5	26.7148
35.5	36.1753
36.5	36.1753
37.5	32.9958
38.5	39.2775
39.5	39.2775
40.5	42.4515
41.5	42.4515
42.5	42.4515
43.5	42.4515
44.5	45.5597
45.5	45.5597
46.5	45.5597
47.5	48.7296
48.5	48.7296
49.5	48.7296
50.5	55.0088
51.5	51.8421
52.5	51.8421
53.5	61.2888
54.5	55.0088
55.5	55.0088
56.5	45.5597
57.5	58.1246
58.5	58.1246
59.5	55.0088
60.5	61.2888
61.5	61.2888
62.5	61.2888
63.5	64.4073
64.5	64.4073
65.5	64.4073

66.5	67.5694
67.5	67.5694
68.5	67.5694
69.5	70.6901
70.5	70.6901
71.5	70.6901
72.5	76.9729
73.5	73.8504
74.5	73.8504
75.5	89.5387
76.5	76.9729
77.5	76.9729
78.5	67.5694
79.5	80.1318
80.5	80.1318
81.5	76.9729
82.5	83.2558
83.5	83.2558
84.5	83.2558
85.5	86.4135
86.5	86.4135
87.5	86.4135
88.5	89.5387
89.5	89.5387
90.5	89.5387
91.5	92.6953
92.5	92.6953
93.5	92.6953
94.5	98.9773
95.5	95.8217
96.5	95.8217
97.5	108.388
98.5	98.9773
99.5	98.9773

- **b**

```
f6[x_, a_] := a x^2;
```

```
f6[x0, a] == y0
```

6421.11 a == 0.131213

```
a = a /. (Solve[f6[x0, a] == y0, a] // Flatten)
```

0.0000204347

```
f6[x_] := f6[x, a]; f6[x]
```

$0.0000204347\, x^2$

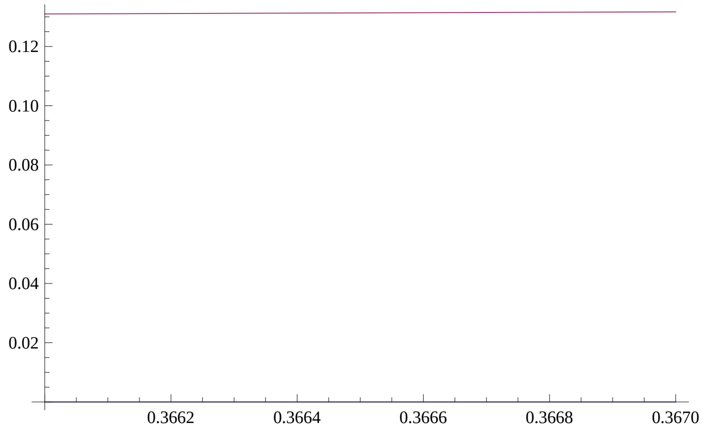
```
a1 = a1 /. (Solve[f6[x1, a1] == y1, a1] // Flatten)
```

0.568052

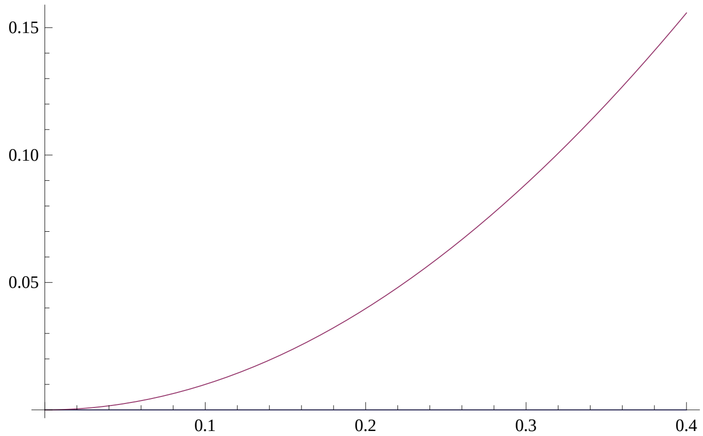
```
f6[x, a1]
```

$0.568052\, x^2$

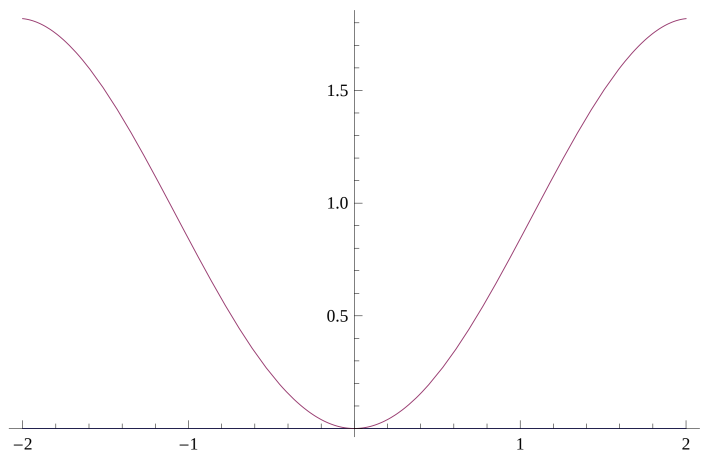
```
Plot[{f6[x], f5[x]}, {x, 0.366, 0.367}]
```




```
Plot[{f6[x], f5[x]}, {x, 0, 0.4}]
```




```
Plot[{f6[x], f5[x]}, {x, -2, 2}]
```

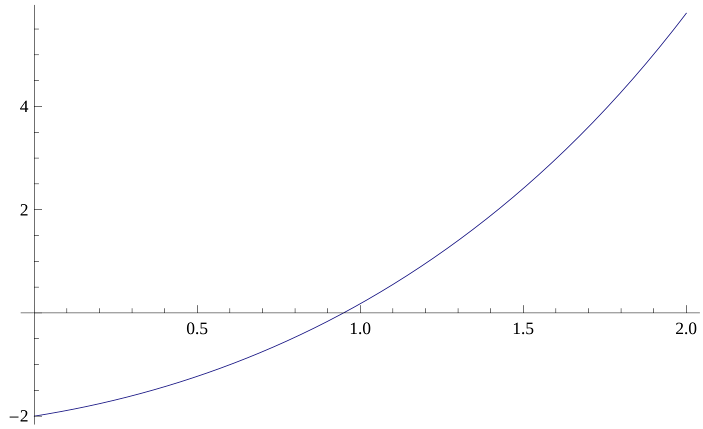


3

```
Remove["Global`*"]
```

▪ a

```
f7[x_] := E^x - Cos[x] - 2;
Plot[f7[x], {x, 0, 2}]
```




```
x[0] = 1.;
x[n_] := x - f7[x] / f7'[x] /. x → x[n - 1];
Table[{n, x[n]}, {n, 0, 4}] // TableForm
```

0	1.
1	0.950002
2	0.948815
3	0.948815
4	0.948815

4

```
Remove["Global`*"]
```

▪ a

```
f8[x_] := 1 / 2 (x - 2) (x - 3) (x + 4);
f8[x] // Expand
```

$$12 - 7 x - \frac{x^2}{2} + \frac{x^3}{2}$$

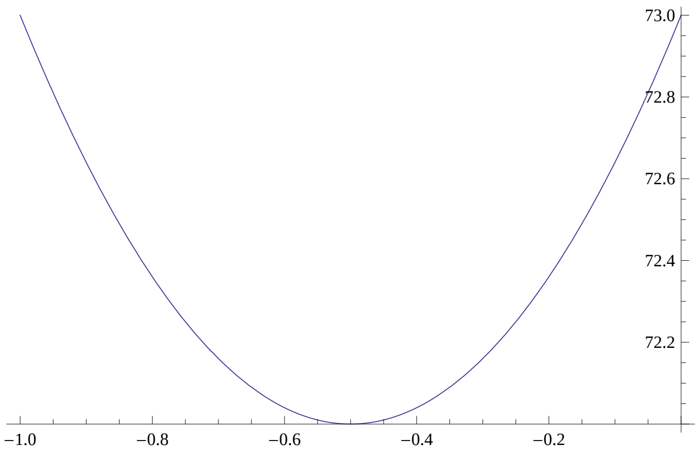
```
f9[x_, b_] := (x - 2) (x - b) (x - 5);
f9[x, b] // Expand
```

$$-10 b + 10 x + 7 b x - 7 x^2 - b x^2 + x^3$$

```
Solve[f8[x] - f9[x, b] == 0, x] // Flatten
```

$$\left\{x \to 2,\ x \to \frac{1}{2}\left(11 + 2 b - \sqrt{73 + 4 b + 4 b^2}\right),\ x \to \frac{1}{2}\left(11 + 2 b + \sqrt{73 + 4 b + 4 b^2}\right)\right\}$$

```
diskr[b_] := 73 + 4 b + 4 b^2;
Plot[diskr[b], {b, -1, 0}]
```



Solve$\left[73 + 4\,b + 4\,b^2 == 0,\ b\right]$

$\left\{\left\{b \to \frac{1}{2}\left(-1 - 6\,i\,\sqrt{2}\right)\right\},\ \left\{b \to \frac{1}{2}\left(-1 + 6\,i\,\sqrt{2}\right)\right\}\right\}$

Solve$\left[2 == \frac{1}{2}\left(11 + 2\,b - \sqrt{73 + 4\,b + 4\,b^2}\right),\ b\right]$

$\{\{b \to 1\}\}$

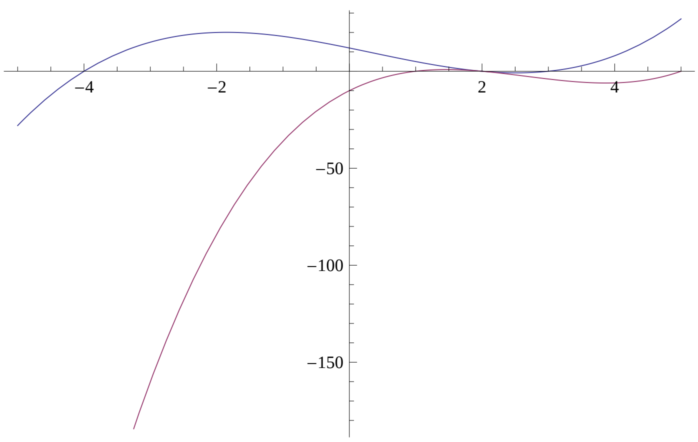
Solve$\left[2 == \frac{1}{2}\left(11 + 2\,b + \sqrt{73 + 4\,b + 4\,b^2}\right),\ b\right]$

$\{\}$

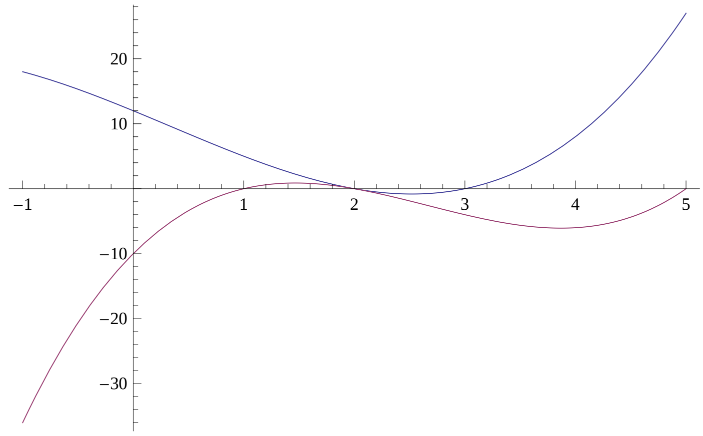
Resultat : Bei x = 2 ist immer ein Schnittpunkt vorhanden.

Da obige Diskriminante immer positiv ist, gibt es immer drei Schnittpunkte, ausser im Falle b = 1, da dort 2 Schnittpunkte zusammenfallen

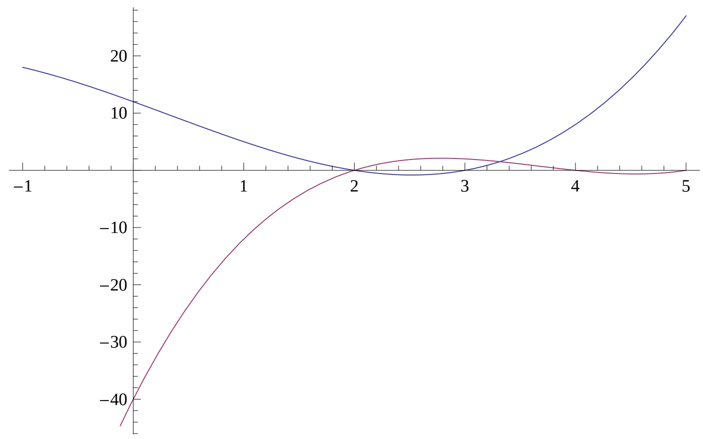
```
Plot[{f8[x], f9[x, 1]}, {x, -5, 5}]
```



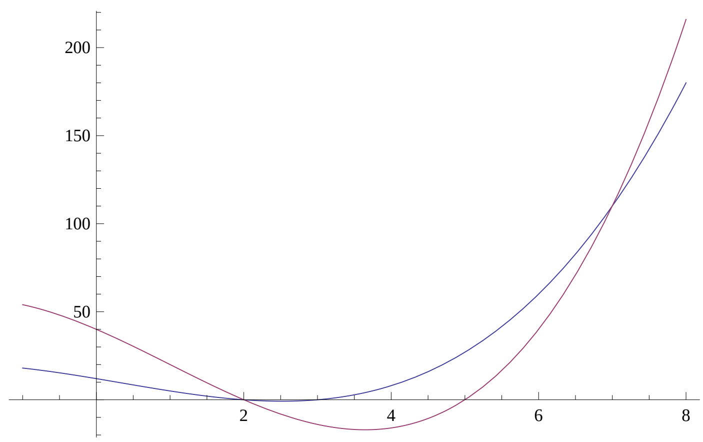
```
Plot[{f8[x], f9[x, 1]}, {x, -1, 5}]
```




```
Plot[{f8[x], f9[x, 4]}, {x, -1, 5}]
```




```
Plot[{f8[x], f9[x, -4]}, {x, -1, 8}]
```



- **b**

```
Solve[f8'[x] == 0, x]
```

$$\left\{\left\{x \to \frac{1}{3}\left(1 - \sqrt{43}\right)\right\}, \left\{x \to \frac{1}{3}\left(1 + \sqrt{43}\right)\right\}\right\}$$

```
N[%]
```

{{x → −1.85248}, {x → 2.51915}}

c

```
Solve[f8''[x] == 0, x]
```

$\{\{x \to \frac{1}{3}\}\}$

```
N[%]
```

{{x → 0.333333}}

d

```
Solve[f8'[x] == Tan[Pi / 4], x]
```

$\{\{x \to -2\}, \{x \to \frac{8}{3}\}\}$

5

```
Remove["Global`*"]
```

a

```
f10[x_] := x^2;
sh[x_] := Sqrt[(2 x) ^2];
su[x_] := Sqrt[x^2 + f10[x] ^2]; su[x]
```

$\sqrt{x^2 + x^4}$

```
so = Solve[sh[x] == su[x], {x}] // Flatten
```

$\{x \to 0, x \to -\sqrt{3}, x \to \sqrt{3}\}$

```
x1 = x /. so[[2]]
```

$-\sqrt{3}$

```
x2 = x /. so[[3]]
```

$\sqrt{3}$

b

```
f10'[x] /. x → x1
```

$-2\sqrt{3}$

```
% // N
```

-3.4641

```
f10'[x] /. x → x2
```

$2\sqrt{3}$

```
% // N
```

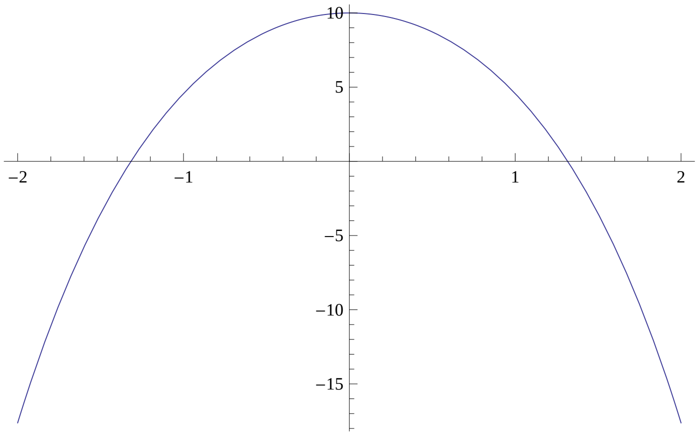
3.4641

6

```
Remove["Global`*"]
```

a

```
f11[x_] := 20 - 10 Cosh[x];
Plot[f11[x], {x, -2, 2}]
```




```
Solve[f11[x] == 0, x]
```

Solve::ifun : Inverse functions are being used by Solve, so
some solutions may not be found; use Reduce for complete solution information. ≫

```
{{x → -ArcCosh[2]}, {x → ArcCosh[2]}}
```

```
N[%]
```

```
{{x → -1.31696}, {x → 1.31696}}
```

```
x1 = x /. FindRoot[f11[x] == 0, {x, -1}]
```

```
-1.31696
```

b

```
DreieckFl[x_] := (x - x1) f11[x] / 2; DreieckFl[x]
```

$\frac{1}{2}$ (1.31696 + x) (20 - 10 Cosh[x])

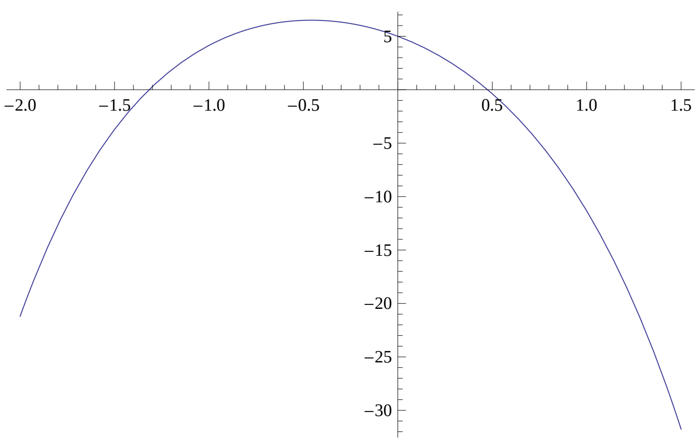
```
FindRoot[ DreieckFl'[x] == 0, {x, 0}]
```

```
{x → 0.475502}
```

```
FindRoot[ DreieckFl'[x] == 0, {x, -1}]
```

```
{x → -1.31696}
```

```
Plot[ DreieckFl'[x], {x, -2, 1.5}]
```



Die Lösung ist x = 0.47550229643917047`

7

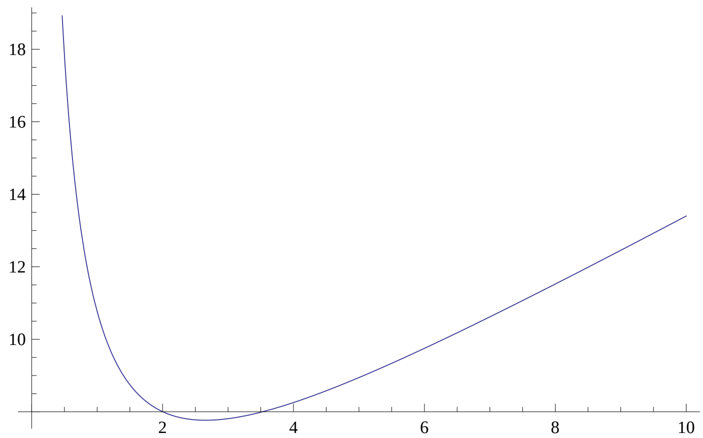
```
Remove["Global`*"]
```

x : 2.5 =(x+3) : y; L^2=(x+3)^2+y^2 = (x^2+2.5^2)/x *(x+3)

```
f12[x_] := Sqrt[x^2 + (2.5)^2] (x + 3) / x; f12[x]
```

$$\frac{(3+x)\sqrt{6.25+x^2}}{x}$$

```
Plot[f12[x], {x, 0, 10}]
```




```
f12'[x]//Simplify
```

$$\frac{-18.75+x^3}{x^2\sqrt{6.25+x^2}}$$

```
so1 = Solve[Evaluate[f12'[x]==0],{x}]
```

{{x → -1.32832 - 2.30072 ⅈ}, {x → -1.32832 + 2.30072 ⅈ}, {x → 2.65665}}

```
xMax = x /. so1[[3]]
```

2.65665

```
Laenge = Sqrt[(xMax^2 + 2.5^2)] / xMax * (xMax + 3)
```

7.76744

```
WinkelInRad = ArcTan[2.5 / xMax]
```

0.75503

```
WinkelInGrad = ArcTan[2.5 / xMax] / Degree
```

43.26

Das ist fast 45 Grad, jedoch nicht exakt 45 Grad!