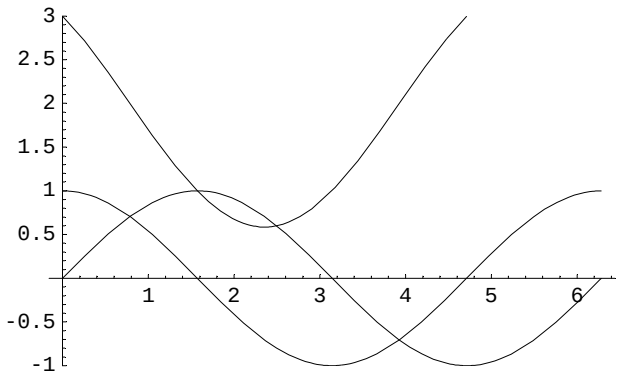
Lösungen

1

a

```
f[x_]:=2+Cos[x]-Sin[x]; a = 0; b= 2 Pi;

Plot[{2+Cos[x]-Sin[x],Cos[x],Sin[x]},{x,0,2 Pi},PlotRange->{-1,3}];
```




```
Integrate[f[x],x]
```

$$2x + \text{Cos}[x] + \text{Sin}[x]$$

```
Integrate[f[x],{x,a,b}]
```

$$4\pi$$

```
N[%]
```

```
12.5664
```

b

```
f[x_]:=x^4/4+x^3/3-3 x^2+2x-4; a = 2; b= -2;

Integrate[f[x],x]
```

$$-4x + x^2 - x^3 + \frac{x^4}{12} + \frac{x^5}{20}$$

```
Integrate[f[x],{x,a,b}]
```

$$\frac{144}{5}$$

```
N[%]
```

```
28.8
```

c

```
f[x_,t_]:=(t^2+2 t) E^x; a = 0; b= -1;
```

```
g[t_]:=Evaluate[Integrate[f[x,t],x]]; g[t]
```

$$e^x\, t\, (2 + t)$$

```
g[t_]:=Evaluate[Integrate[f[x,t],{x,a,b}]]; g[t]
```

$$-\frac{(-1 + e)\, t\, (2 + t)}{e}$$

```
D[g[t],{t}]
```

$$-\frac{(-1 + e)\, t}{e} - \frac{(-1 + e)\, (2 + t)}{e}$$

```
Solve[Evaluate[D[g[t],{t}]]==0,{t}]
```

$$\{\{t \to -1\}\}$$

d

```
Remove[a]
```

```
f[x_]:= a Cos[2 x Pi-Pi/2]; u = 0; b = Pi;
```

```
Integrate[f[x],x]
```

$$-\frac{a\, \text{Cos}[2\, \pi\, x]}{2\, \pi}$$

```
Integrate[f[x],{x,u,b}]
```

$$\frac{a\, \text{Sin}[\pi^2]^2}{\pi}$$

```
N[%]
```

```
0.058938 a
```

e

```
Remove[u]
```

```
f[x_]:= x^3 Log[u x]; a = 1; b= E;
```

```
Integrate[f[x],x]
```

$$-\frac{x^4}{16} + \frac{1}{4}\, x^4\, \text{Log}[u\, x]$$

```
Integrate[f[x],{x,a,b}]
```

$$\frac{1}{16}\left(1+3\,e^4+4\left(-1+e^4\right)\text{Log}[u]\right)$$

```
N[%]
```

```
0.0625 (164.794 + 214.393 Log[u])
```

```
%//Expand
```

```
10.2997 + 13.3995 Log[u]
```

f

```
f[x_]:=3x/x/(x-1)/(x+1); a = 2; b= 4;
```

```
Integrate[f[x],x]
```

$$3\left(\frac{1}{2}\text{Log}[-1+x]-\frac{1}{2}\text{Log}[1+x]\right)$$

```
Integrate[f[x],{x,a,b}]
```

$$\frac{3}{2}\text{Log}\left[\frac{9}{5}\right]$$

```
N[%]
```

```
0.88168
```

```
Integrate[f[x],{x,a,b}]/.Log->log
```

$$\frac{3}{2}\,\text{log}\left[\frac{9}{5}\right]$$

$$\frac{3}{10}\left(-5\,\text{log}[5]+5\,\text{log}[19]+\sqrt{5}\,\text{log}\left[\frac{1}{19}\left(21+4\sqrt{5}\right)\right]\right)\text{ /. }a\_\,\text{log}[x\_]+b\_\,\text{log}[y\_]+c\_\,\text{log}[z\_]\to\text{log}[x\text{\^{}}(a)\;y\text{\^{}}b\;z\text{\^{}}c]$$

$$\frac{3}{10}\,\text{log}\left[25\,19^{4-\sqrt{5}}\left(21+4\sqrt{5}\right)^{\sqrt{5}}\right]$$

g

```
f[x_]:=x E^(-x^2); a = 0; b= 1;
```

```
Integrate[f[x],x]
```

$$-\frac{e^{-x^2}}{2}$$

```
Integrate[f[x],{x,a,b}]
```

$$\frac{-1+e}{2e}$$

```
N[%]
```

```
0.31606
```

2

```
Remove["Global`*"]
```

```
f[x_]:=2 E^(-(x-1)^2); a = 0; b= 3/2;
```

```
Integrate[f[x],x]
```

$$\sqrt{\pi}\ \text{Erf}[-1+x]$$

```
Integrate[f[x],{x,a,b}]
```

$$\sqrt{\pi}\ \left(\text{Erf}\left[\frac{1}{2}\right]+\text{Erf}[1]\right)$$

```
N[%]
```

```
2.41621
```

a

```
n=8
```

```
8
```

```
xt[k_]:= k(b-a)/n
```

```
sR = Sum[f[xt[k]] (b-a)/n,{k,1,n}]//N
```

```
2.48437
```

b

```
sT = f[xt[a]]/2 (b-a)/n+Sum[f[xt[k]] (b-a)/n,{k,1,n-1}]+ f[xt[b]]/2 (b-a)/n // N
```

```
2.37314
```

c

```
sT-sR
```

```
-0.111221
```

d

```
n=100;
Sum[f[xt[k]] (b-a)/n,{k,1,n}]//N
```

```
2.42232
```

```
f[xt[a]]/2 (b-a)/n+Sum[f[xt[k]] (b-a)/n,{k,1,n-1}]+ f[xt[b]]/2 (b-a)/n // N
```

```
2.41024
```

3

```
Remove["Global`*"]
```

a

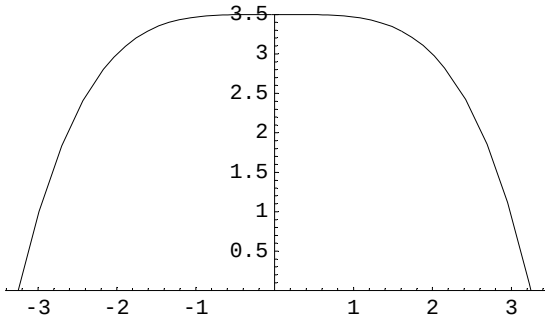
```
h = 3.5;
b = 6.5;
l = 20000;
f[x_]:= a x^4+h;

solv = Solve[f[b/2]==0,{a}] // Flatten
```

$\{a \to -0.0313715\}$

```
f[x_]:= a x^4+h /.solv; f[x]
```

$3.5 - 0.0313715\, x^4$

```
Plot[f[x],{x,-b/2,b/2},AspectRatio->Automatic,PlotRange->{0,h}];
```



b

```
A1 = Integrate[f[x],{x,-b/2,b/2}]
```

```
18.2
```

```
A1 = Integrate[f[x],{x,-b/2,b/2}]//N
```

```
18.2
```

```
r1 = Integrate[f[x],{x,-b/2,b/2}] l
```

```
364000.
```

c

```
a = a/.solv
```

$$-0.0313715$$

```
f[x_]:= a x^4 + h1; f[x]
```

$$h1 - 0.0313715\, x^4$$

```
solv = Solve[f[b1/2]==0,{h1}] // Flatten
```

$$\{h1 \to 0.00196072\, b1^4\}$$

```
f[x_]:= a x^4+h1  /. solv; f[x]
```

$$0.00196072\, b1^4 - 0.0313715\, x^4$$

```
Integrate[f[x],{x,-b1/2,b1/2}] == 2 A1
```

$$0.00156857\, b1^5 == 36.4$$

```
solv1 = Solve[Integrate[f[x],{x,-b1/2,b1/2}] == 2 A1,{b1}] // Flatten
```

$$\{b1 \to -6.04056 - 4.38872\, i,\ b1 \to -6.04056 + 4.38872\, i,\ b1 \to 2.30729 - 7.1011\, i,\ b1 \to 2.30729 + 7.1011\, i,\ b1 \to 7.46654\}$$

```
solv1[[5]]
```

$$b1 \to 7.46654$$

```
b1 = b1 /. solv1[[5]]
```

$$7.46654$$

```
solv
```

$$\{h1 \to 6.09385\}$$

4

```
Remove["Global`*"]
```

a

```
f[x_,c_]:= Integrate[t^3 /4+-3 t,t]+c /.t->x;  f[x,c]
```

$$c - \frac{3\, x^2}{2} + \frac{x^4}{16}$$

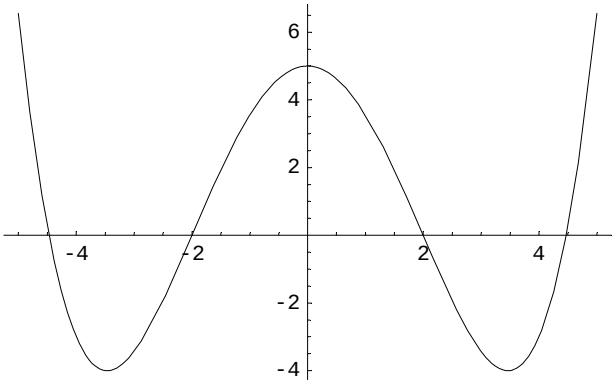
```
f[0,c]==5
```

$$c == 5$$

```
f[x_]:= f[x,5]; f[x]
```

$$5 - \frac{3\,x^2}{2} + \frac{x^4}{16}$$

```
Plot[f[x],{x,-5,5}];
```



b

```
Solve[Evaluate[D[f[x],x]==0],{x}]
```

$$\left\{\{x \to 0\}, \left\{x \to -2\sqrt{3}\right\}, \left\{x \to 2\sqrt{3}\right\}\right\}$$

```
N[%]
```

```
{{x → 0.}, {x → -3.4641}, {x → 3.4641}}
```

c

```
Solve[f[x]==0,{x}]
```

$$\left\{\{x \to -2\}, \{x \to 2\}, \left\{x \to -2\sqrt{5}\right\}, \left\{x \to 2\sqrt{5}\right\}\right\}$$

```
N[%]
```

```
{{x → -2.}, {x → 2.}, {x → -4.47214}, {x → 4.47214}}
```

d

```
Solve[Evaluate[D[f[x],{x,2}]==0],{x}]
```

```
{{x → -2}, {x → 2}}
```

e

```
Integrate[f[x],{x,-2,2}]
```

$$\frac{64}{5}$$

```
N[%]
```

```
12.8
```

5

```
Remove["Global`*"]
```

```
Integrate[3x/x/(x-1)/(x+1),{x,2,4}]
```

$$\frac{3}{2}\,\text{Log}\left[\frac{9}{5}\right]$$

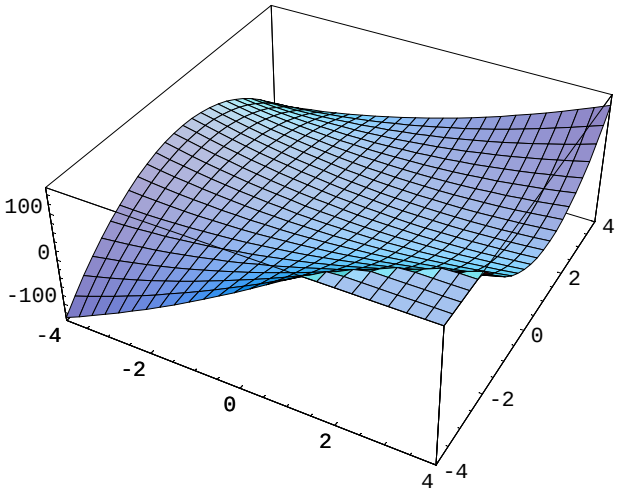
```
f[x_,y_]:=2 (x-y)^2+3 x y^2-5 (2 y+x)
```

```
f[x,y]//TeXForm
```

```
2 (x-y)^2+3 x y^2-5 (x+2 y)
```

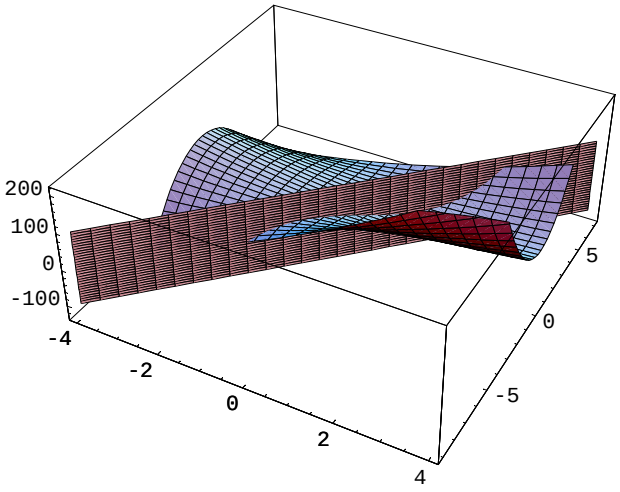
```
f[x_,y_,z_]:=f[x,y]
```

```
p1=Plot3D[f[x,y],{x,-4,4},{y,-4,4}];
```




```
p2=ParametricPlot3D[{x,2x-1,z},{x,-4,4},{z,-100,100}];
```

```
Show[p1,p2];
```



a

```
<<Calculus`VectorAnalysis`
```

```
gf=Grad[f[x,y,z], Cartesian[x, y, z]]
```

$$\{-5+4\,(x-y)+3\,y^2,\ -10-4\,(x-y)+6\,x\,y,\ 0\}$$

b

```
Solve[Evaluate[Grad[f[x,y,z], Cartesian[x, y, z]]]==0,{x,y,z}]
```

$$\left\{\left\{x\to\frac{1}{4},\ y\to 2\right\},\ \left\{x\to-\sqrt{\frac{5}{3}},\ y\to-\sqrt{\frac{5}{3}}\right\},\ \left\{x\to\sqrt{\frac{5}{3}},\ y\to\sqrt{\frac{5}{3}}\right\}\right\}$$

```
Solve[Evaluate[Grad[f[x,y,z], Cartesian[x, y, z]]]==0,{x,y,z}]//N
```

$$\{\{x\to 0.25,\ y\to 2.\},\ \{x\to -1.29099,\ y\to -1.29099\},\ \{x\to 1.29099,\ y\to 1.29099\}\}$$

c

```
g[x_,y_]:=2x-1-y
```

```
g[x_,y_,z_]:=g[x,y]
```

```
gg=Grad[g[x,y,z], Cartesian[x, y, z]]
```

$$\{2,\ -1,\ 0\}$$

```
Solve[{gf[[1]]==λ gg[[1]],gf[[2]]==λ gg[[2]],g[x,y]==0},{λ,x,y}]
```

$$\left\{\left\{\lambda\to\frac{1}{27}\left(109-7\sqrt{259}\right),\ x\to\frac{1}{18}\left(5+\sqrt{259}\right),\ y\to\frac{1}{9}\left(-4+\sqrt{259}\right)\right\},\right.$$
$$\left.\left\{\lambda\to\frac{1}{27}\left(109+7\sqrt{259}\right),\ x\to\frac{1}{18}\left(5-\sqrt{259}\right),\ y\to\frac{1}{9}\left(-4-\sqrt{259}\right)\right\}\right\}$$

```
Solve[{gf[[1]]==λ gg[[1]],gf[[2]]==λ gg[[2]],g[x,y]==0},{λ,x,y}]//N
```

```
{{λ → -0.135346, x → 1.17186, y → 1.34372}, {λ → 8.20942, x → -0.616304, y → -2.23261}}
```