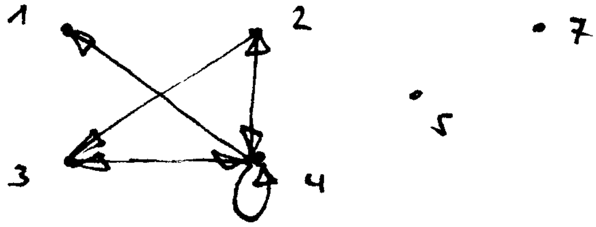
3 $\quad f: \mathbb{R} \mapsto \mathbb{R}, \quad f(x) = \begin{cases} -x & x \in \mathbb{Q} \\ 2x & x \in \mathbb{R} \setminus \mathbb{Q} \end{cases}$

Sei $f_1(x) = -x$, $D_{f_1} = \mathbb{Q}$ $\qquad y = f_1(x) = -x \xleftrightarrow{\text{bij}} x \quad (x = -y)$
(inj. und surj.)

$f_2(x) = 2x$, $D_{f_2} = \mathbb{R} \setminus \mathbb{Q}$ $\qquad y = f_2(x) = 2x \xleftrightarrow{\text{bij}} x \quad (x = \frac{y}{2})$

$\leadsto \quad f(x) = \begin{cases} f_1(x) & x \in \mathbb{Q} \\ f_2(x) & x \in \mathbb{R} \setminus \mathbb{Q} \end{cases}$ überall bijektiv (inj. u. surj.)

4 Es fehlen:



$\leadsto$ 7 Pfeile!

5 $|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C|$

$105 = 81 + 24 + 35 - 11 - 15 - 8 + x$

$\Rightarrow \quad x = 3$ $\qquad$ (keine Schnittmenge ist mächtiger als eine Obermenge $\leadsto$ o.k.)

6 $(\mathbb{Z}_5, +, \cdot)$ Körper

$\overset{[2]}{\cancel{[4]}} \cdot ([3] \cdot [x] - [2]) = \cancel{[2]} \cdot [x]$

$\underbrace{[6]}_{=[1]} [x] - [4] = [x]$

$\Rightarrow \quad \cancel{[x]} - [4] = \cancel{[x]} \Rightarrow -[4] = [0]$: W!

$\Rightarrow \quad \mathbb{L} = \{ \}$

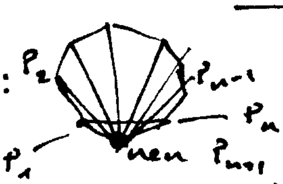
7 Verankerung: $n = 3$

0 Diagonalen! $D_3 = \frac{3 \cdot \overbrace{(3-3)}^{0}}{2} = 0$ ✓

Vererbung: Ind. Vor.: $D_k = \frac{k(k-3)}{2} \quad (n = k)$

Ind. Beh.: $D_{k+1} = \frac{(k+1)(k-2)}{2} \quad (n = k+1)$

Bew.:



Neuer Punkt P_{n+1}: Neue Diag $\overline{P_{n+1}P_2}, \dots, \overline{P_{n+1}P_{n-1}}$
sowie $\overline{P_1 P_n}$ $\leadsto$ $n-1$ Stück

$D_k + \underbrace{(k-1)}_{\text{neue}} = \frac{k(k-3)}{2} + k - 1 = \frac{k^2 - 3k + 2k - 2}{2} = \frac{k^2 - k - 2}{2} = \frac{(k+1)(k-2)}{2} = D_{k+1}$ ✓